AN ANALYSIS OF OIL SAMPLE DATA OBTAINED FROM AIRCRAFT ENGINES BY SPECTOMETRY.

by

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United States Naval Postgraduate School

THESIS

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Obtained from Aircraft Engines by Spectrometry

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Major, United States Marine Corps
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ABSTRACT

The spectrometric oil analysis program as applied to naval aviation was developed as a portion of the overall aviation safety program of the U. S. Navy. The equipment and techniques have been refined, and the program has been steadily expanded since its inception in 1955. The value of this system in determining densities of microscopic particles of certain oil-wetted wear metals in samples of oil extracted from aircraft engines has proved to be helpful in predicting incipient engine failure. In this study data relating to both reciprocating and jet engine models was analyzed in an attempt to determine which of the following elements provided significant information regarding the internal condition of the engine: aluminum, iron, chromium, silver, magnesium, nickel, copper, and silicon.

Multiple and simple linear regression analyses and correlation techniques were applied in order to determine the mathematical model which corresponded most closely to the data compiled.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF DRAWINGS

I. INTRODUCTION

The United States Navy Spectrometric Oil Analysis Program had a modest beginning in 1955 at the Naval Air Rework Facility, Naval Air Station, Pensacola, Florida. From this beginning the program has grown to the extent that it now enjoys a substantial amount of credit for the success of the Navy's overall compaign for aviation safety. The degree of success attained was largely due to the ability of certain types of spectrometers to detect changes in amounts of microscopic particles of metals suspended in an oil solution. Experience has shown that early identification of incipient engine failure may be recognized through proper interpretation of the analysis of rates of particle build-up in the oil samples.

Current state-of-the-art spectrometry can measure concentrations of all of the important wear metals produced in an oil-lubricated mechanical system. Since metallic ions emit characteristic light spectra when vaporized in an electric arc, a unique spectrum for each metal is obtained. When the intensity of the spectrum is measured against a relative standard, the result is an estimate of the quantity of the metal present in the sample being analyzed.

With the spectrometric analysis of the sample completed, the only remaining task involves interpretation of data. The accession of modern unit record equipment has allowed the oil analysis program to expand and to build a sufficiently large data base from which statistical inferences may be drawn. This collection of data and experience led toward the establishment of threshold limits for certain elements on each particular engine model. These limits have been delineated for use as a guide in determining whether or not an engine is in need of overhaul.

To assist engine mechanics in the performance of their maintenance duties, the analyst in the central laboratory at Pensacola can often pinpoint a potential trouble spot in an engine by noting which particular metal has exceeded its threshold. For instance, aluminum is often used in reciprocating engines for parts such as pistons, cylinder heads, and oil pump impellers. Copper is almost always used for bushings and intake valve guides, and nickel is used for exhaust valve guides. No single standard list can be compiled which would be appropriate for all engines. However, each model has a list of standardized parts. This standardization within models is what allows the analyst at the central laboratory to detect not only probable, impending trouble but also to predict the engine part which will most likely fail.

II. CONCEPT OF THE STUDY

This study will be concerned primarily with analysis of data collected during the period 1 July 1967 - 30 September 1967 at the Naval Air Station, Pensacola, Florida. Attention will be focused upon a reciprocating engine model produced by the Pratt and Whitney Aircraft Division, United Aircraft Corporation. The specific model is the R2000-6. The data for three different models of jet engines was also examined. The jet engine analysis covered models J52-P-6, J60-P-6, and J33-A-24. Since the results relating to jet engine data differed from those relating to the reciprocating model a discussion of general results for the jet models is presented separately in Appendix H. The procedure for spectrometric analysis of oil samples is identical for both jet and reciprocating engines. Whenever a sample is analyzed by the spectrometer, the data is automatically recorded. Each sample provides a data point consisting of a number of components. Fifteen of the components which are of interest to this study are

(1) Engine model number

(2) Engine serial number

(3) Date oil sample removed from engine

(4) Hours since engine overhaul (engine operating hours)

(5) Hours since engine oil change (engine operating hours)

(6) Density of aluminum (parts per million)

(7) Density of iron (parts per million)

(8) Density of chromium (parts per million)

(9) Density of silver (parts per million)

(10) Density of copper (parts per million)

(11) Density of tin (parts per million)

(12) Density of magnesium (parts per million)

(13) Density of lead (parts per million)

(14) Density of nickel (parts per million)

(15) Density of silicon (parts per million)

Throughout this study we shall refer to several of the above-mentioned elements less frequently than to others. For instance, silicon is the only non-metallic element in the data. It is used primarily as an indicator of the amount of sand that a reciprocating engine has ingested from the atmosphere. Analysis for silicon contamination in jet engines has become meaningless due to the increased use of silicone additives in jet engine oils.

Reference to a specific serial number of a specific model engine will be by the digit numbers (1), (2),...,(26) rather than by the full six digit serial numbers. This simplifies the notation throughout the study. Appendix A may be referred to for determining which serial number is related to which digit number.

Practically speaking, the variables which can be measured without error are the hours since engine overhaul and the hours since engine oil change. These two variables have been designated the independent variables, and the densities of the ten elements have been designated as the dependent variables. The purpose of this study is the investigation of relationships which may exist between items (4) and (5) of the list of components, and the last ten items.

Since little information of an analytical nature exists, the observed data was plotted on a scatter diagram to aid in determining what, if any, functional relationships exist. With acceptance of the assumption of normality, regression analysis provided the natural technique for analytically uncovering linear trends between the variables, and for expressing any existing functional relationships in mathematical form. By this method the best relation among the variables was determined. Further analysis using correlation techniques showed the strength with which the variables were associated. When preliminary investigations into the data showed a rather high correlation between the two independent variables, a stepwise regression procedure was conducted in order to determine the more desirable course of action: continuation of the analysis using multiple linear regression techniques, or simplifying the analysis by employing simple linear regression.

III. PROCEDURE

Computations and computer plots for this study were conducted on the IBM 360 computer at the Naval Postgraduate School, Monterey. Initial scatter diagrams were not encouraging. The plot of each element versus each of the two independent variables was reproduced in Appendix B. These plots depicted the entire set of data points for all 26 engines for model R2000-6. Although the data did not appear to have strong linear properties, the plots were interesting enough to invite a more detailed investigation, particularly the trends exhibited by aluminum, iron, silver, and copper.

A sampling of five of the engines was selected for individual plots and the data thus obtained graphed separately. The results are shown in Appendix C. These results proved to be far more encouraging since they reinforced the initial impression that the four elements; aluminum, iron, copper, and silver showed definite linear trends.

Multiple linear regression techniques were applied to the raw data. The predicted value, (y'), was expressed by the equation

$$y' = a + b_1 x_1 + b_2 x_2$$

Here x_1 is hours since engine overhaul; and x_2 is hours since engine oil change. These are the variables affecting y which is the density in parts per million of the element being predicted.

Estimates for the partial regression coefficients, b_1 and b_2, and the constant term, a, were computed using the procedure outlined in Ostle.[1]

While analyzing the 26 engines of model R2000-6, it was considered desirable to determine if all 26 engines could be treated as if they came from the same population. In addition to the normality assumption, if the further assumption of equal variances for all 26 engines proved acceptable, the data could possibly be pooled and the engines could be handled as one. However, due to the suspect nature of the initial scatter diagrams, the assumption of homogeneity of variances was not made. Therefore, the test selected to handle this problem was the one developed by Bartlett.[2] In order to test for homogeneity of variances the null hypothesis was

$$H_o : \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_{26}^2 = \sigma^2$$

Bartlett's statistic has been shown to be distributed under H_o approximately as chi square with K-1 degrees of freedom.

The test was conducted at the $\alpha = .05$ level for the elements aluminum, iron, chromium, silver, magnesium, nickel, copper, and silicon. With the exception of chromium, the null hypothesis was rejected. Not only had the equal variances test failed, but it was also found that the multiple linear regression had indicated that the partial regression coefficient b_1 would often have a

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[1]Ostle, B., Statistics In Research, 2nd Ed., p. 177, Iowa State, 1963.

[2]Brownlee, K. A., Statistical Theory and Methodology In Science And Engineering, p. 225, Wiley, 1960.

positive value while b_2 would have a negative value and vice versa. Since the evidence was overwhelmingly against homogeneity of variances for the multiple linear regression situation, it was deemed advisable to conduct a stepwise regression analysis to determine which of the two independent variables was the better predictor. A further benefit from this analysis was that an objective evaluation could also be made to determine the relative merit of retaining the other variable in the equation.

The stepwise procedure used was a linear regression program designated BIMEDØ2R. This program is one of a series of library programs designed by the Health Sciences Computing Facility, University of California, Los Angeles. A sequence of multiple linear regression equations is computed in a stepwise manner. At each step one variable is added to the regression equation. The variable added is the one which makes the greatest reduction in the error sum of squares. This is the same as saying that it is the variable which has the highest partial correlation with the dependent variable partialed on the variables which have already been added to the equation. Since this analysis had only two independent variables, the first variable which was entered into the equation "explained more of the variance" than did the second variable.

Stepwise regressions were conducted on each of the elements for which the Naval Air Rework Facility had previously established threshold limits. For engine model R2000-6 limits have been established for aluminum, iron, chromium, silver, copper, nickel, and silicon. Results of the stepwise regression for the four elements aluminum, iron, copper, and silver showed that hours since engine oil change appeared as the first variable entered in the regression equation far more often than did hours since engine overhaul. The results obtained from the remaining elements were inconclusive as to which of the two independent variables was more consistent in explaining variance. For these elements hours since engine overhaul appeared as the first variable entered in the step-wise procedure about as often as did hours since engine oil change. Appendix D contains the specific outcome for each of the 26 engines.

With the stepwise regression completed, a test of the hypothesis that the slope of the regression line was zero was conducted.

The first test had as the null hypothesis:

$$H_{01} : \beta_{1i} = 0 \qquad ; i = 1, 2, \ldots, 26$$

To accomplish this, the following ratio was formed from the Analysis of Variance Table:

$$\frac{\text{mean square due to } b_1 | a}{\text{residual mean square}}$$

This ratio follows an F- distribution with $\nu_1 = 1$ and $\nu_2 = n_i - 2$ degrees of freedom where n_i is the number of observations taken on the i-th engine.

The next test had as the hypothesis:

$$H_{02} : \beta_{2i} = 0 \qquad ; i = 1, 2, \ldots, 26$$

To accomplish this test, the following ratio was formed from the Analysis of Variance Table:

$$\frac{\text{mean square due to } b_2 | a, b_1}{\text{residual mean square}}$$

This ratio follows the F- distribution with $\nu_1 = 1$ and $\nu_2 = n_i-3$ degrees of freedom. The results of testing the foregoing hypothesis at the $\alpha = .05$ level are contained in Appendix E.

The ratio for testing H_{01} assesses the significance of the reduction in the residual sum of squares gained by entering the first variable in the equation. The notation $b_1|a$ is used to indicate that the sums of squares associated with the coefficients are obtained sequentially, Likewise, the ratio for testing H_{02} is used to determine if any significance should be attached to the additional reduction in the residual sum of squares achieved by entering b_2 into the equation after the sums of squares for a and b_1 have been computed. Thus, the notation $b_2|a,b_1$ was used.

As part of the stepwise procedure the estimated multiple correlation coefficients were computed at the completion of each step. The addition of the second variable into the multiple linear regression equation caused an increase in the correlation coefficient of 0.1000 or more in only the number of engines shown in Table I.

The relatively small increase in correlation is not considered significant in view of the additional increase in the computational effort required to effect the higher correlation.

Element	Number of Engines
Aluminum	4
Iron	2
Copper	4
Silver	5
Chromium	3
Magnesium	9
Nickel	7
Silicon	6

TABLE I

Increases of 0.10 or More in Multiple Correlation Coefficient Caused by Addition of the Second Independent Variable to the Equation

IV. RESULTS

A significant result obtained from the analysis of the stepwise regression showed that hours since engine oil change was the better predictor of the two independent variables. Furthermore when the results of the tests for zero-slope were scanned, it was noted that the hypothesis

$$H_{02} : \beta_{2i} = 0 \qquad ; i = 1, 2, \ldots, 26$$

was accepted more often than the hypothesis

$$H_{01} : \beta_{1i} = 0 \qquad ; i = 1, 2, \ldots, 26$$

This acceptance of hypotheses held true for each of the metallic elements in the analysis. For the element silicon the number of engines for which H_{02} was accepted proved to be equal to the number for which H_{01} was accepted. This result, when considered with the multiple correlation coefficient results, indicated that not only did the addition of the second variable to the regression add little to the correlation coefficient but, in some cases, its addition actually tended to nullify some of the effect of positive slope provided by the first variable. That is, it was found that instances occurred wherein one independent variable was positively correlated with the dependent variable, and the other independent variable was negatively correlated with the dependent variable. With this evidence established the decision was made to drop the variable hours since engine overhaul from the analysis and to proceed using only the remaining independent variable hours since engine oil change.

The computer program in Appendix G was written to perform a simple linear regression on each of the 26 engines for each of the following elements: aluminum, iron, chromium, silver, magnesium, nickel, copper, and silicon. The possibility of constructing one single regression equation which could be used for all 26 engines had already been ruled out by Bartlett's Test for homogeneity of variances. Therefore, the conclusion that each engine should be treated individually was firmly established.

A final task which the program in Appendix G was designed to perform was that of establishing a prediction interval of the independent variable x. Such an interval was to be predicted for each engine and for each element.

Now, since the regression was reduced to a simple linear form, the expected value of y which corresponds to a given $x = X$ is calculated as

$$y' = a + bX$$

Using this predictor for y we can employ the formula for a $100\,(1-\alpha)\%$ prediction interval for y which is presented in Ostle.[3]

Appendix F displays the results obtained from a hypothetical value of $x = 150$ for a 95 per cent prediction interval around y. Since y represented the parts per million density for the element under consideration, the low end of the interval was set equal to zero at any time that the predicted low end dropped below zero. A negative number of parts per million for an element would be

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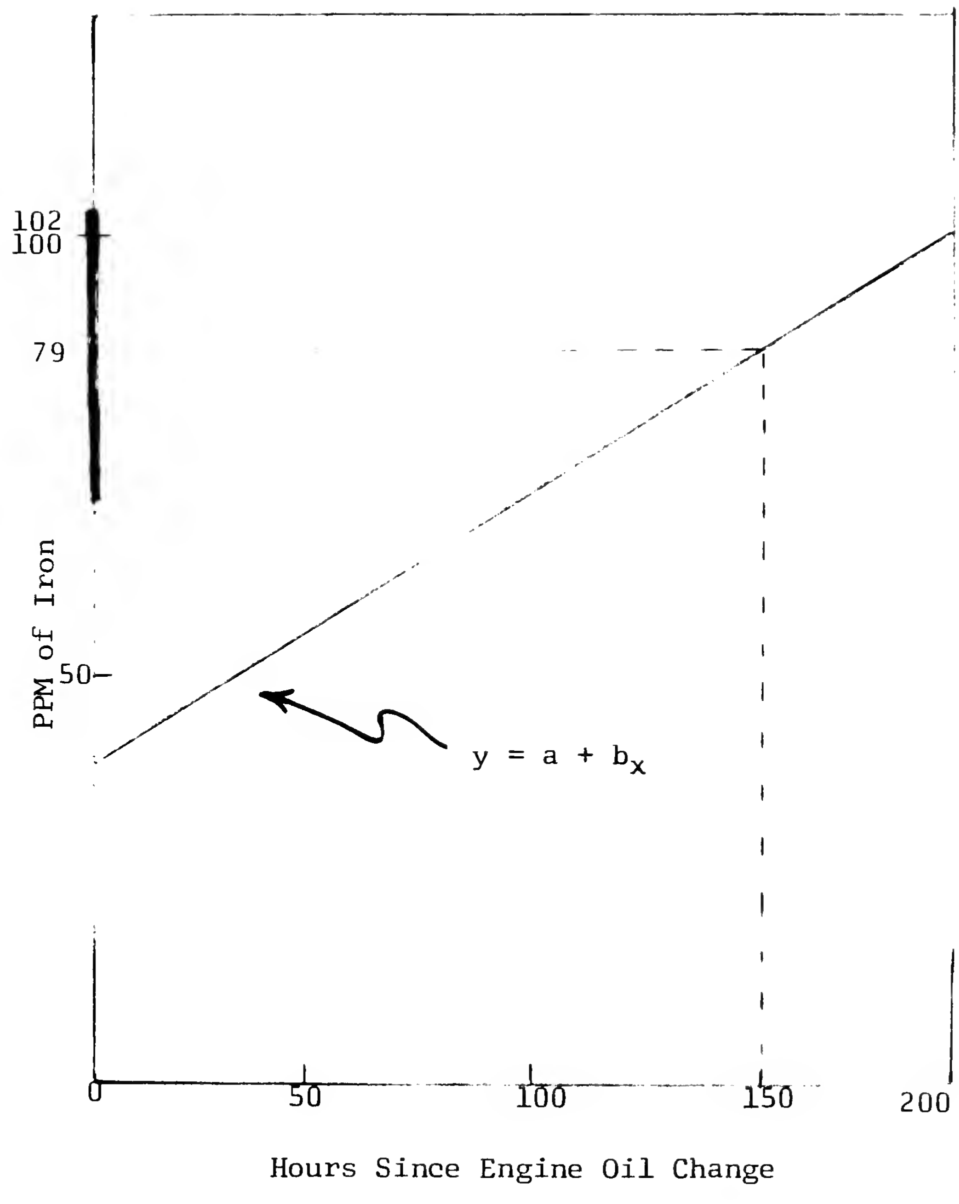
[3]Ostle, B., Statistics In Research, 2nd Ed., p. 170, Iowa State, 1963.

meaningless. Also, since the spectrometer in use at the Naval Air Station, Pensacola, measures densities of the elements in integer values, the low end values were truncated to integer values and the high end values were rounded up to the next highest integer if they were composed of any decimal fraction. An illustration is shown in Drawing I.



DRAWING 1

Example of Prediction Interval for an Oil Sample Extracted from an Engine at a Point In Time 150 Hours Since Engine Oil Change

V. CONCLUSIONS

The policy pertaining to engine model R2000-6 requires collection of data for both of the independent variables hours since engine oil change as well as hours since engine overhaul. In addition to this, data is also collected for aluminum, iron, chromium, silver, copper, nickel, and silicon. It appears that virtually all of the information relating to this model engine is contained in the data for aluminum, iron, copper, silver, and hours since engine oil change. These five items provide a noticeably linear trend whereas chromium, nickel, silicon, and hours since engine overhaul appear to possess poorer predictive powers.

Each engine has its own characteristic build-up rate for each element; therefore data collection, storage, and manipulation must, of necessity, be accomplished separately for each particular serial number of each model engine. When a new engine serial number is introduced into the system, there can be very little information of a predictive nature which can be fed back to the operating unit until a sufficient number of data points has been recorded. This study utilized serially numbered engines only if there were eight or more data points with which to work. Once an engine has provided enough data points to characterize its build-up rate of parts per million for each element, any new reading can be examined against its prediction interval and appropriate action taken. Normally, appropriate action could be

(1) If the actual y value is within the prediction interval for all elements, the point is added to the data bank and the prediction mechanism uses this data point along with all

previous points to update the regression equation. Obviously, the more data points collected on a particular engine, the better should be the prediction.

(2) If the actual y value is above the predicted high end of the interval for any one or more of the elements associated with a particular model engine, then the operating unit is notified with an appropriate recommendation. The data point is then added to the data bank and treated as in (1) above.

A further study should be conducted using data collected over a longer period of time such as a three-year period. The results obtained from such a study should be much more conclusive than these findings which have been based on data collected over a three-month period.

APPENDIX A

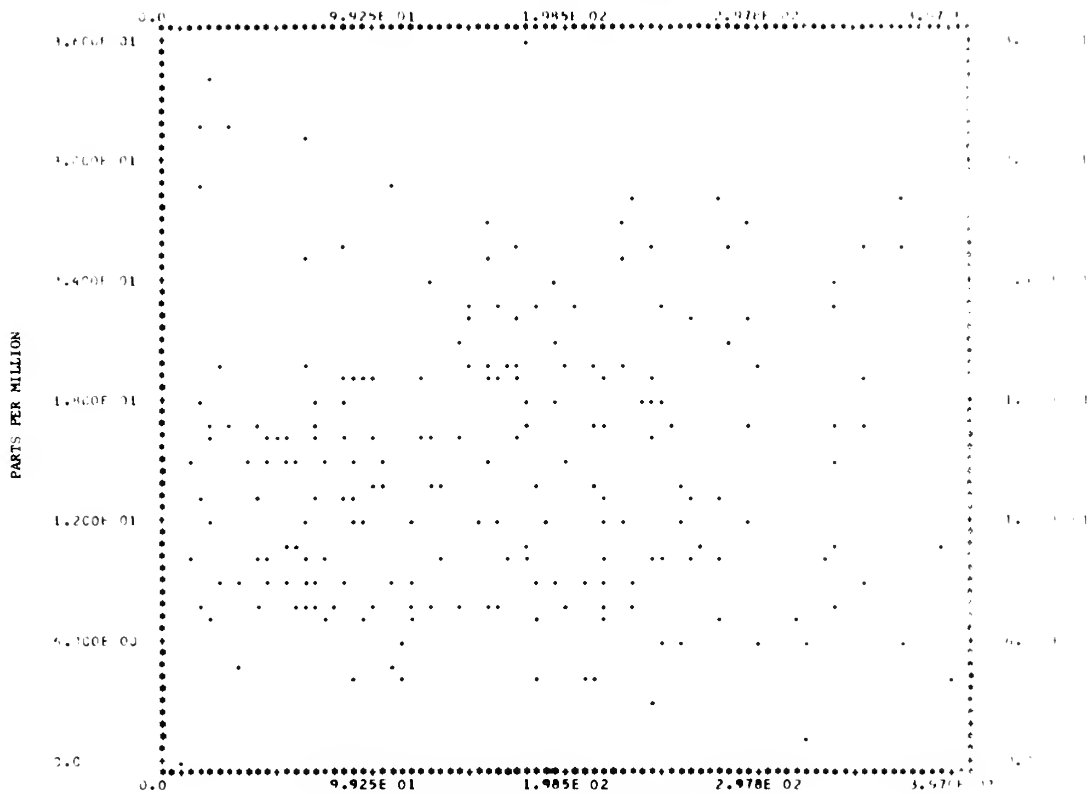
SERIAL NUMBERS OF THE R2000-6 MODEL AIRCRAFT ENGINES

Engine Number	Serial Number
1	101196
2	103105
3	104166
4	105005
5	105246
6	106505
7	106666
8	106681
9	106690
10	106707
11	106797
12	106802
13	107486
14	107935
15	108003
16	108372
17	108458
18	108461
19	108553
20	108616
21	108919
22	700002
23	700130
24	700163
25	702269
26	702489

APPENDIX B

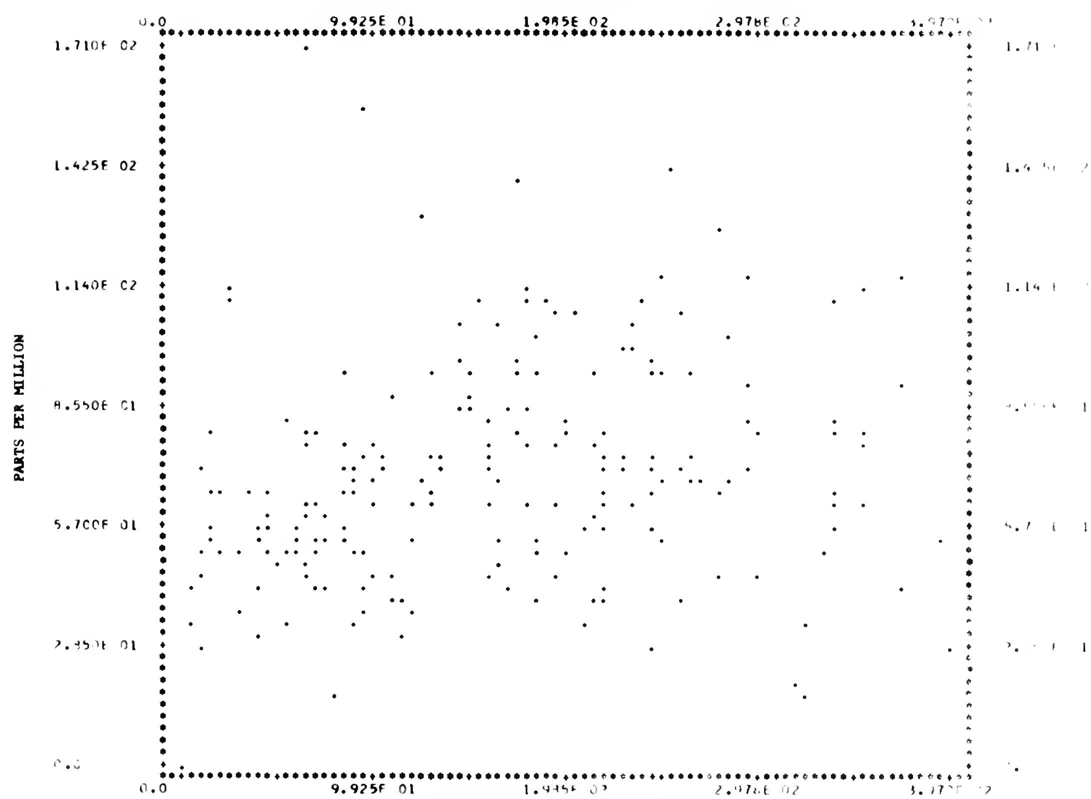
SCATTER DIAGRAMS OF RAW DATA

DATA FOR ALUMINUM



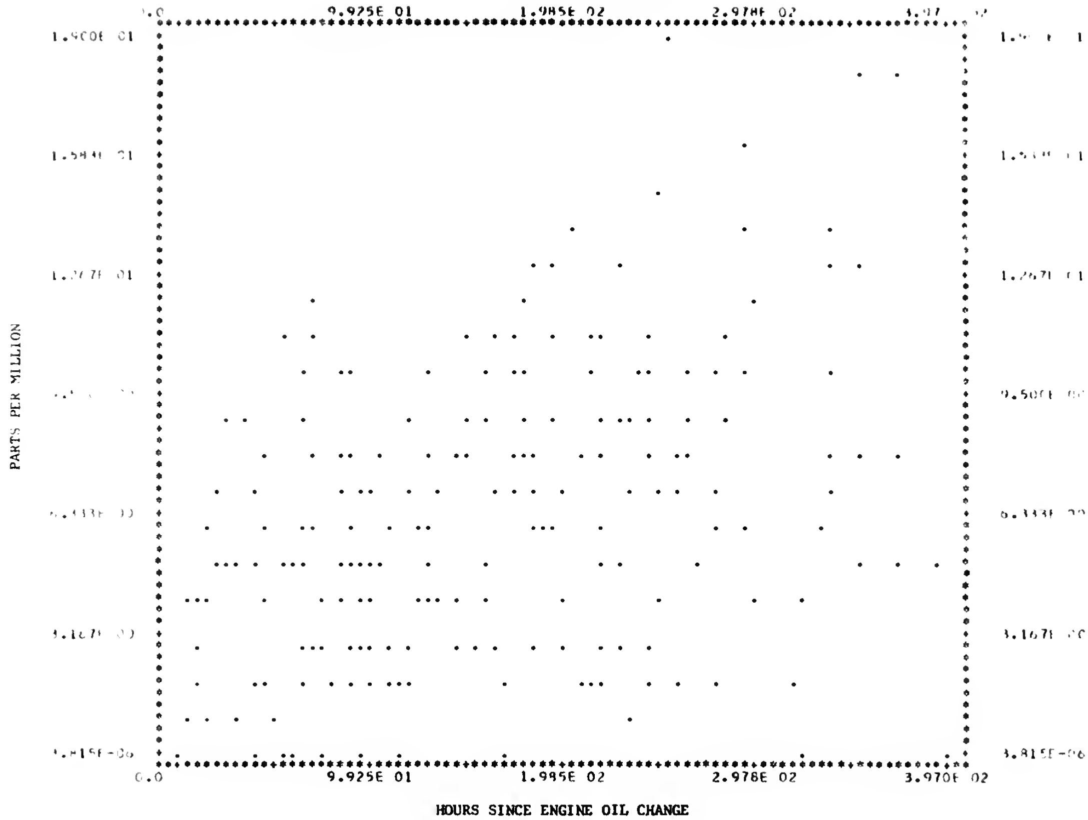
HOURS SINCE ENGINE OIL CHANGE

DATA FOR IRON

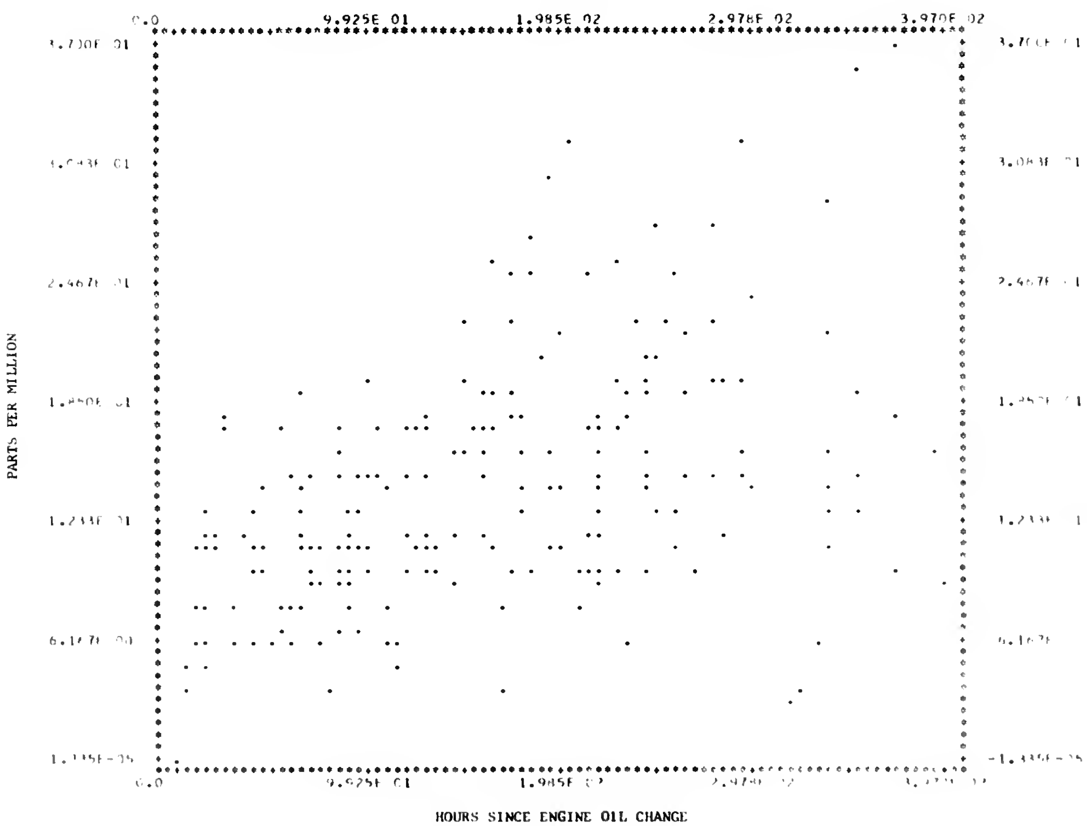


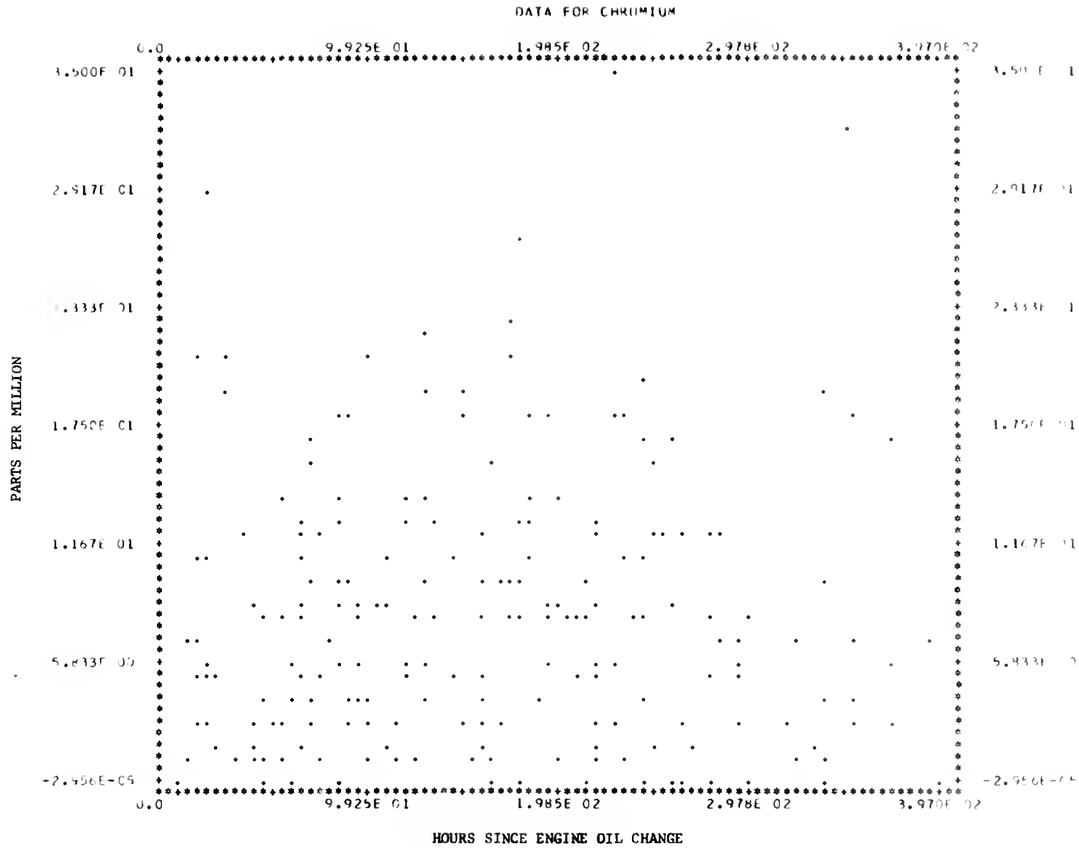
HOURS SINCE ENGINE OIL CHANGE

DATA FOR SILVER



DATA FOR COPPER




DATA FOR CHRUMIUM
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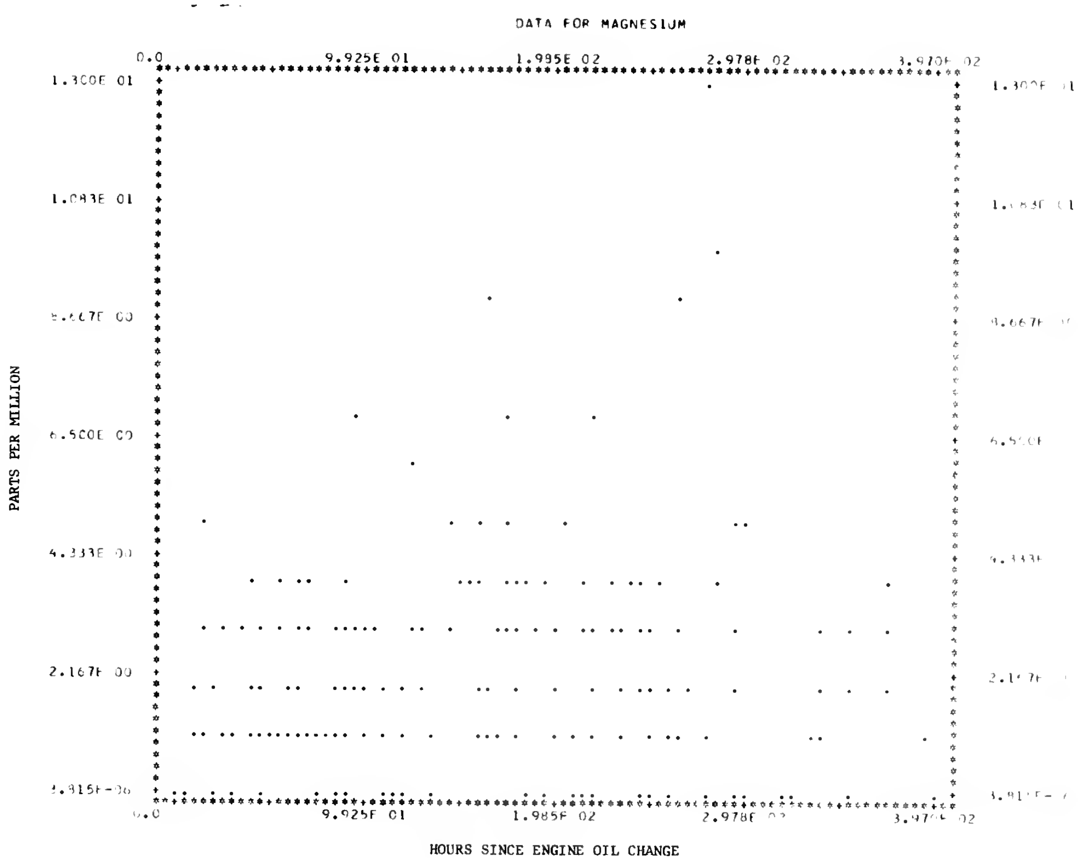
1.985E 02
2.978E 02
3.970E 02
3.500E 01
2.917E 01
2.333E 01
1.750E 01
1.167E 01
5.833E 00
-2.956E-05
PARTS PER MILLION
HOURS SINCE ENGINE OIL CHANGE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1.985E 02
2.978E 02
3.970E 02
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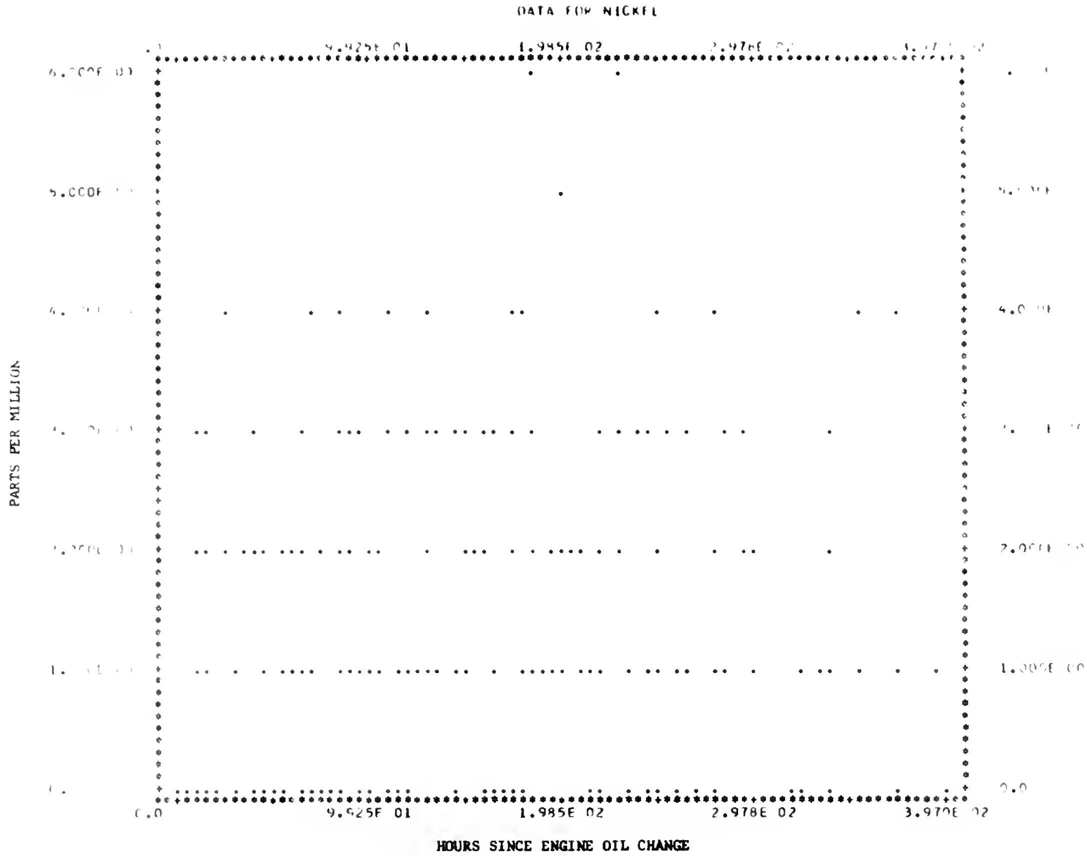
1.083E 01
8.667E 00
6.500E 00
4.333E 00
2.167E 00
PARTS PER MILLION
HOURS SINCE ENGINE OIL CHANGE


DATA FOR NICKEL
PARTS PER MILLION
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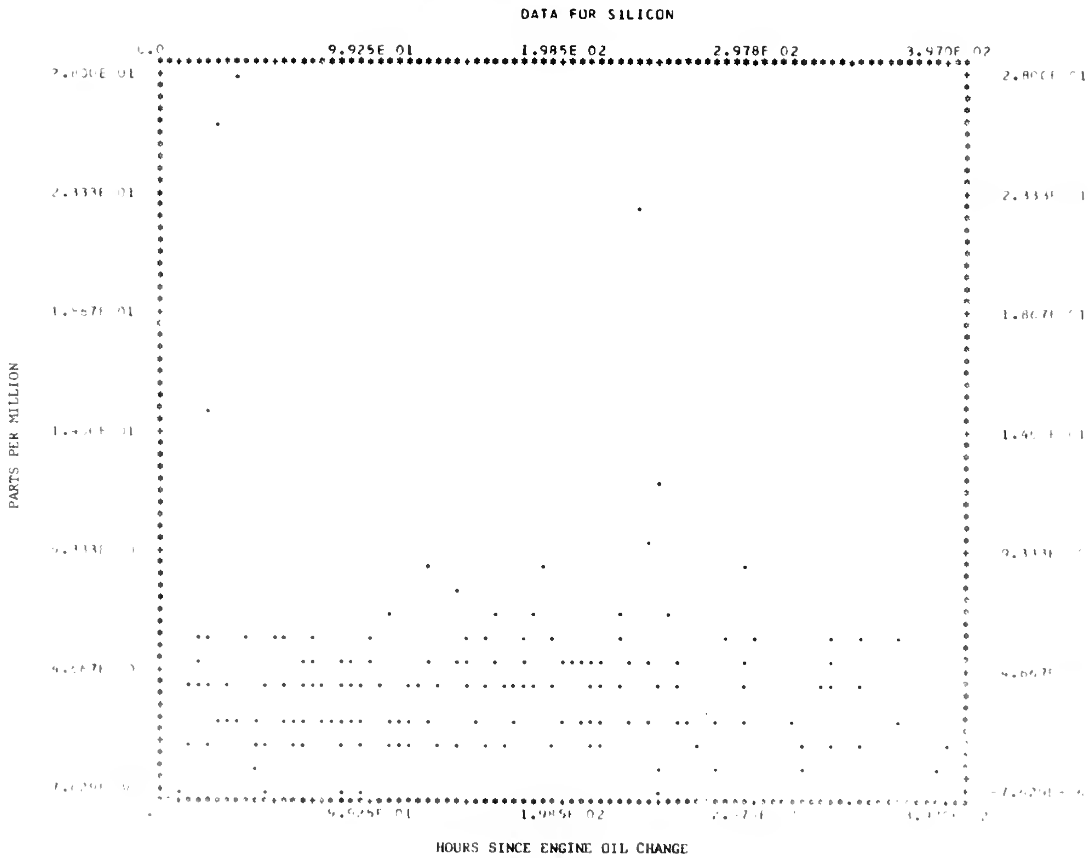
2.978E 02
HOURS SINCE ENGINE OIL CHANGE


DATA FOR SILICON
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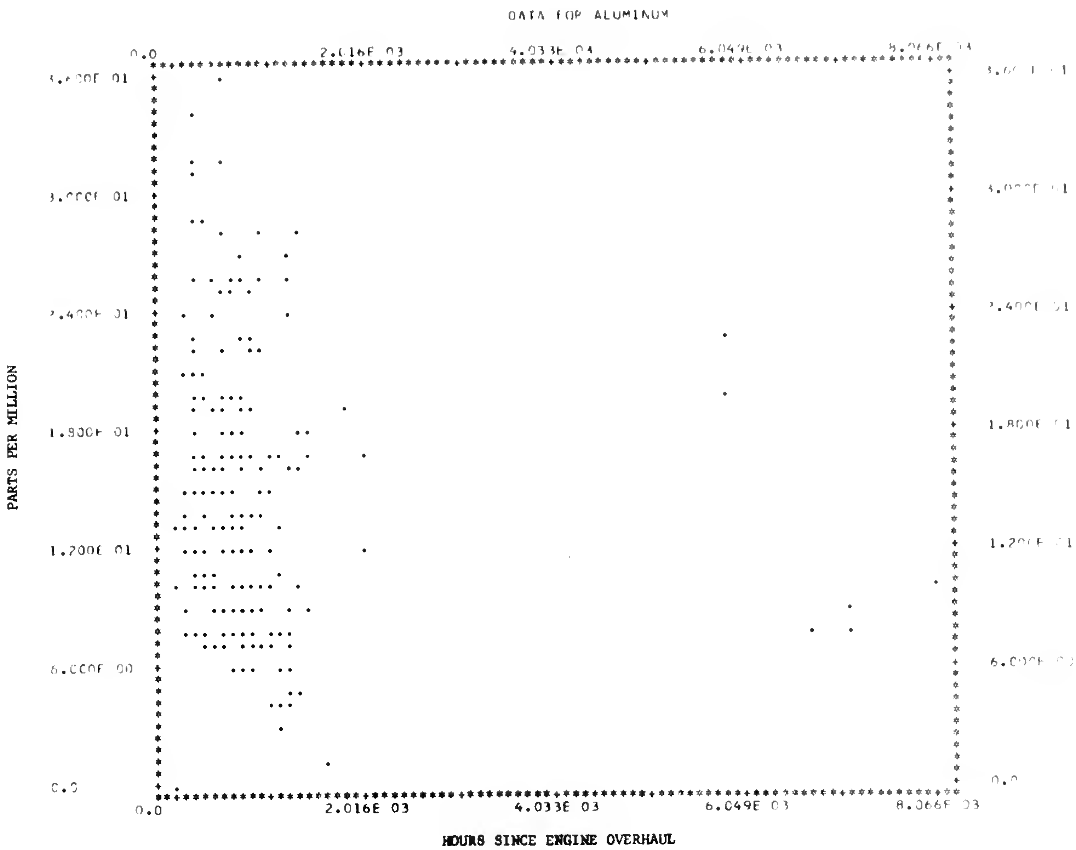
2.978F 02
2.800E 01
2.333E 01
PARTS PER MILLION
HOURS SINCE ENGINE OIL CHANGE


DATA FOR ALUMINUM
PARTS PER MILLION
HOURS SINCE ENGINE OVERHAUL
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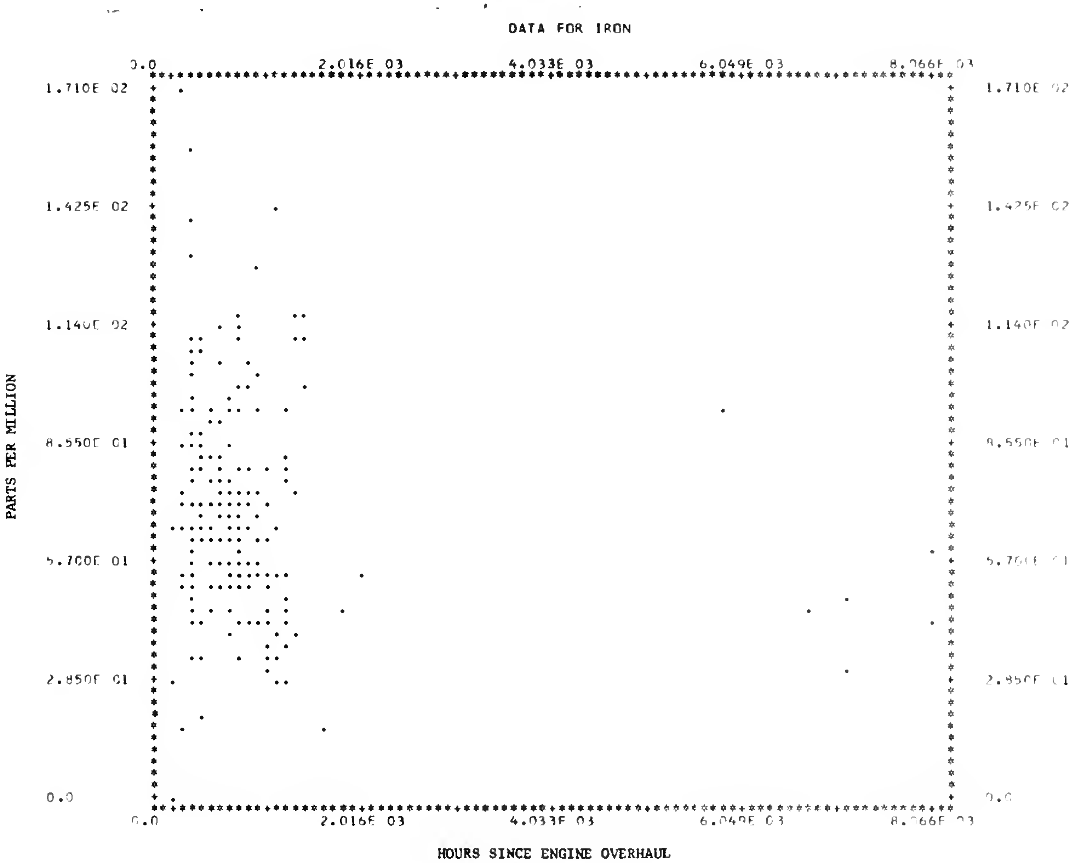
8.066E 03
1.800E 01
1.200E 01
0.0


DATA FOR IRON
PARTS PER MILLION
HOURS SINCE ENGINE OVERHAUL
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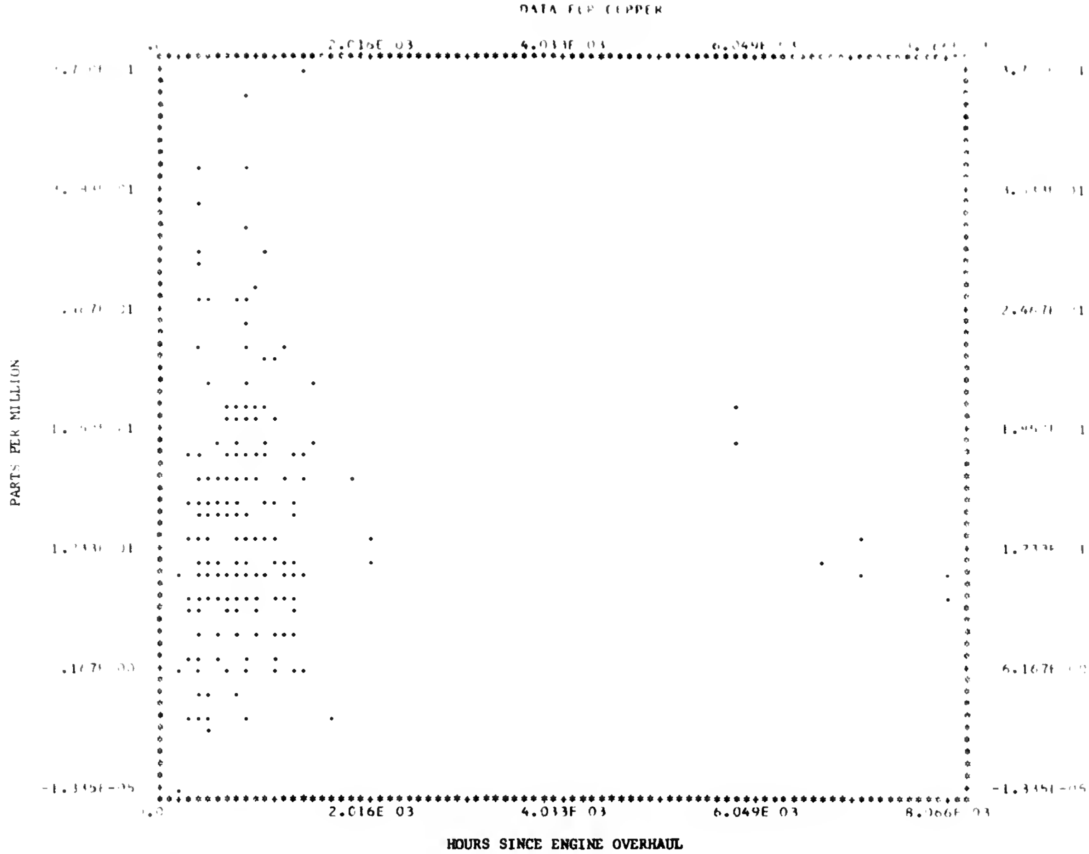
8.550E 01
2.850E 01
0.0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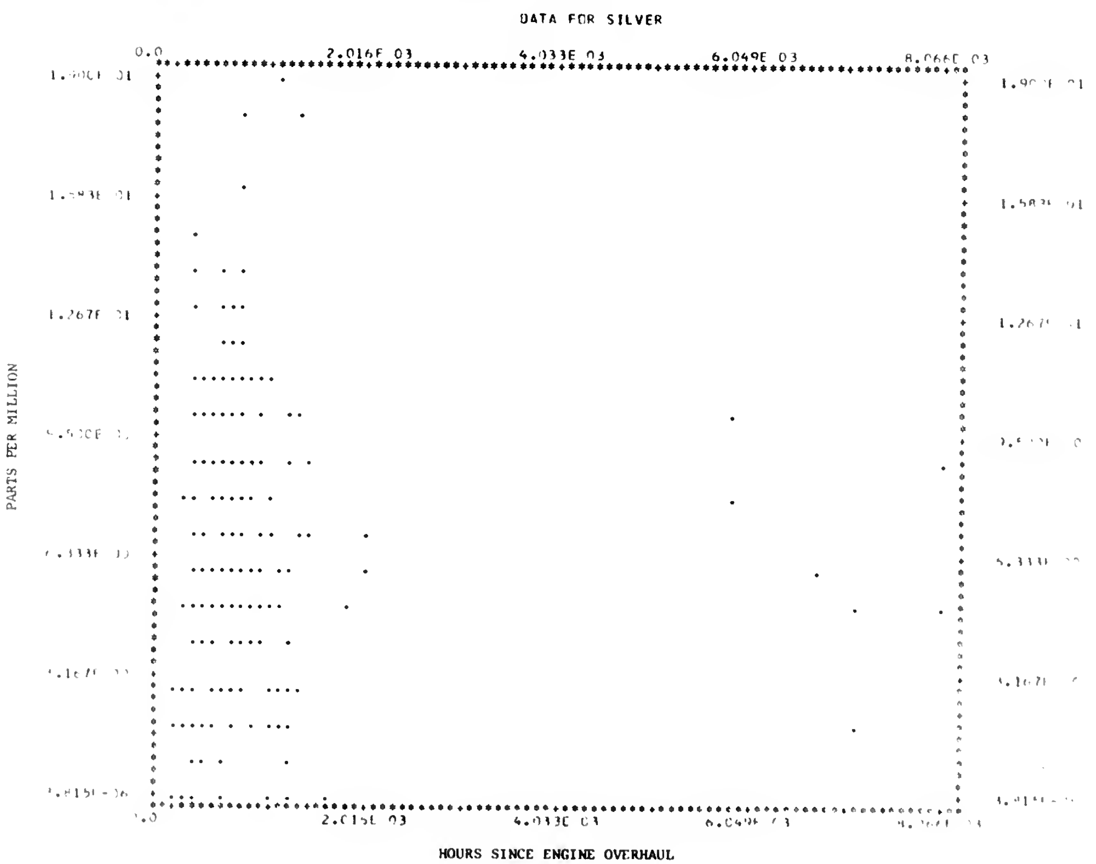
4.033E 03
6.049E 03
8.066E 03
PARTS PER MILLION
HOURS SINCE ENGINE OVERHAUL


DATA FOR SILVER
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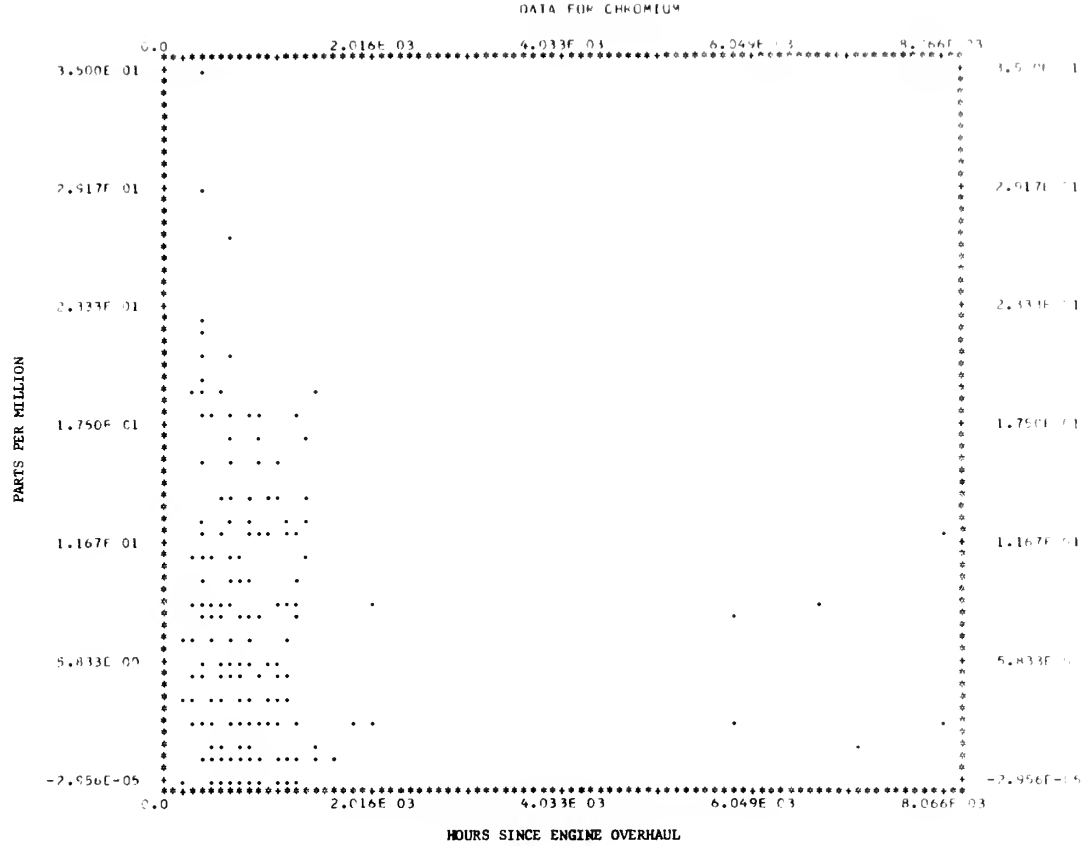
4.033E 03
6.049E 03
8.066E 03
1.267E 01
PARTS PER MILLION
HOURS SINCE ENGINE OVERHAUL


DATA FOR CHROMIUM
PARTS PER MILLION
3.500E 01
2.917E 01
2.333E 01
1.750E 01
1.167E 01
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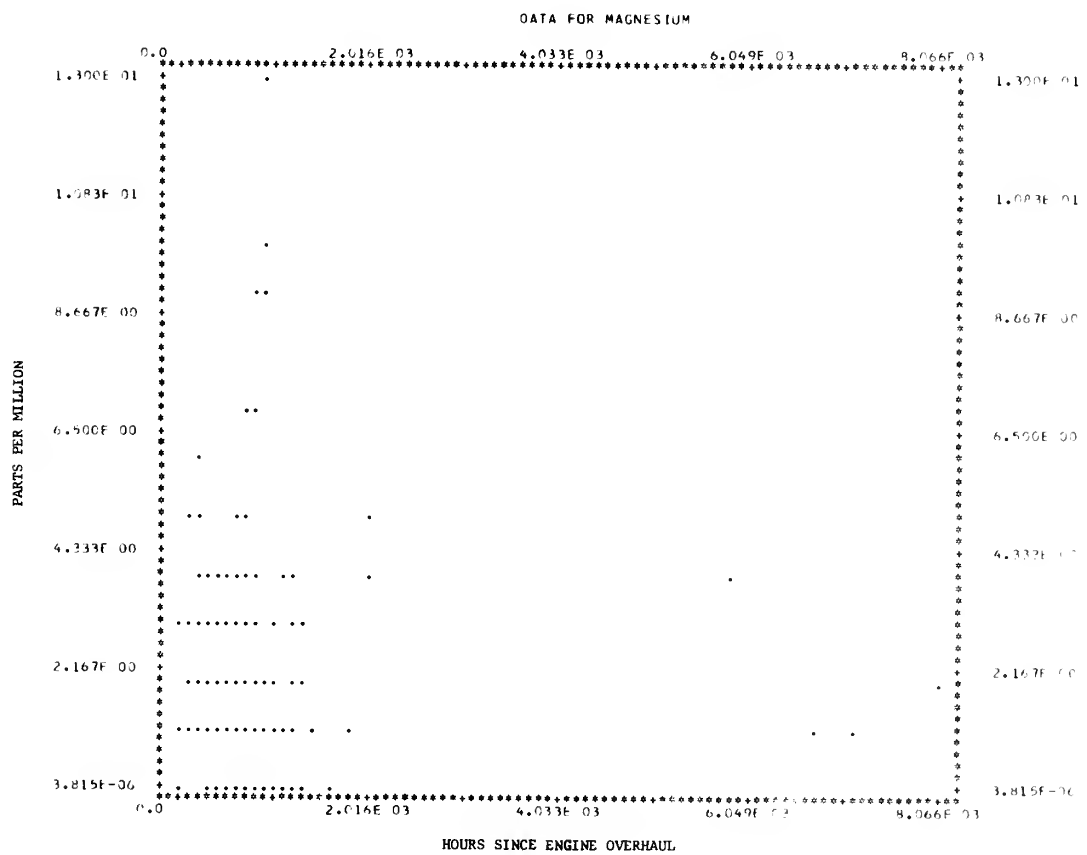
-2.956E-05
0.0
2.016E 03
4.033E 03
6.049E 03
8.066E 03
HOURS SINCE ENGINE OVERHAUL


DATA FOR MAGNESIUM
PARTS PER MILLION
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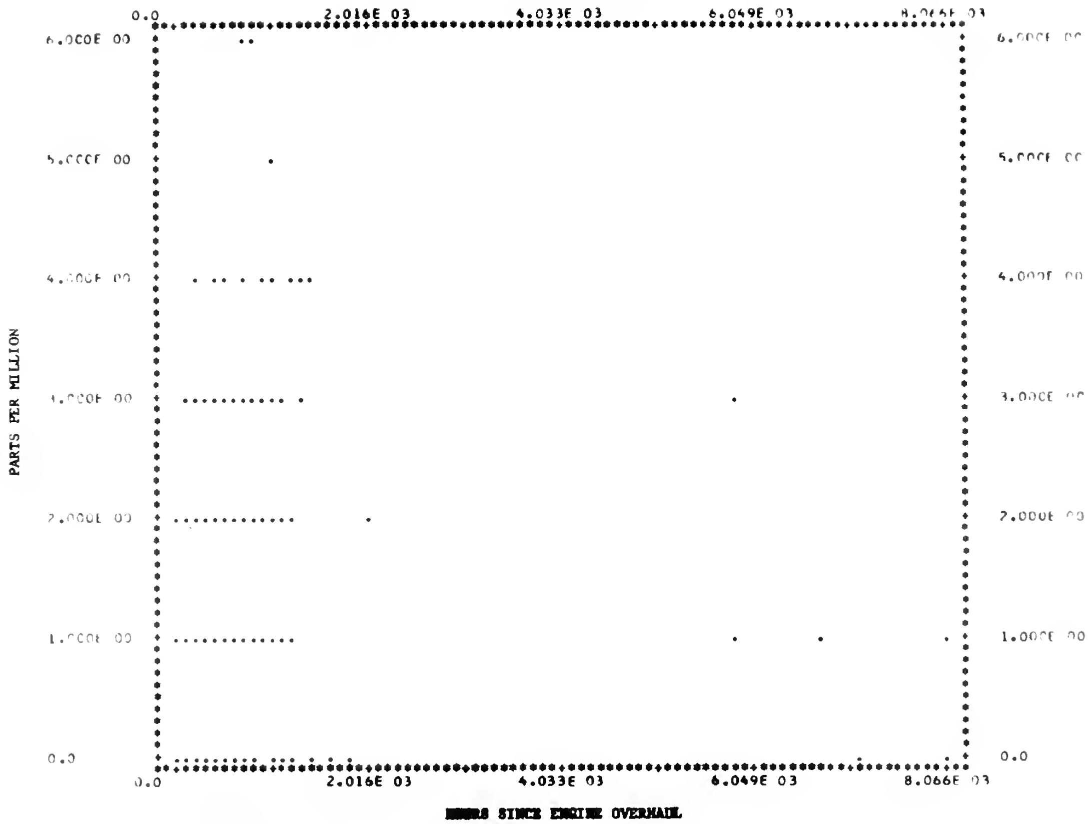
1.083E 01
8.667E 00
6.500E 00
4.333E 00
2.167E 00
3.815E-06
0.0
2.016E 03
4.033E 03
6.049E 03
8.066E 03
HOURS SINCE ENGINE OVERHAUL

DATA FOR NICKEL



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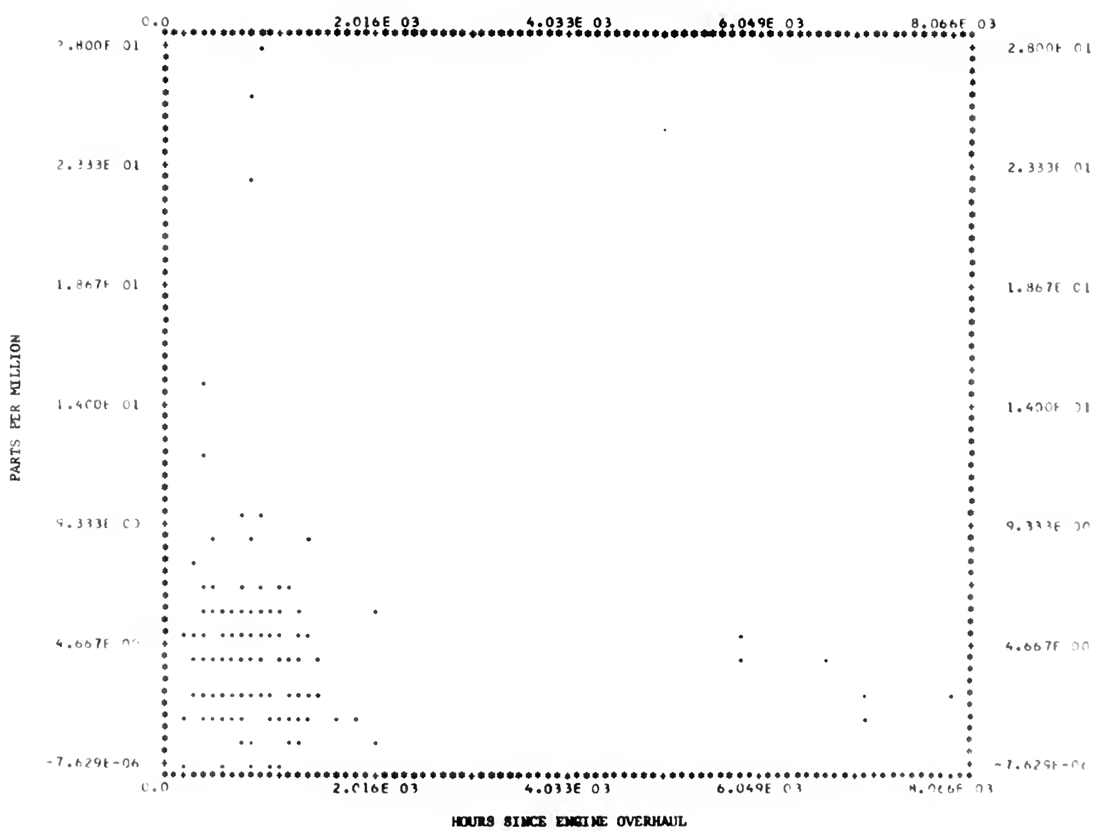
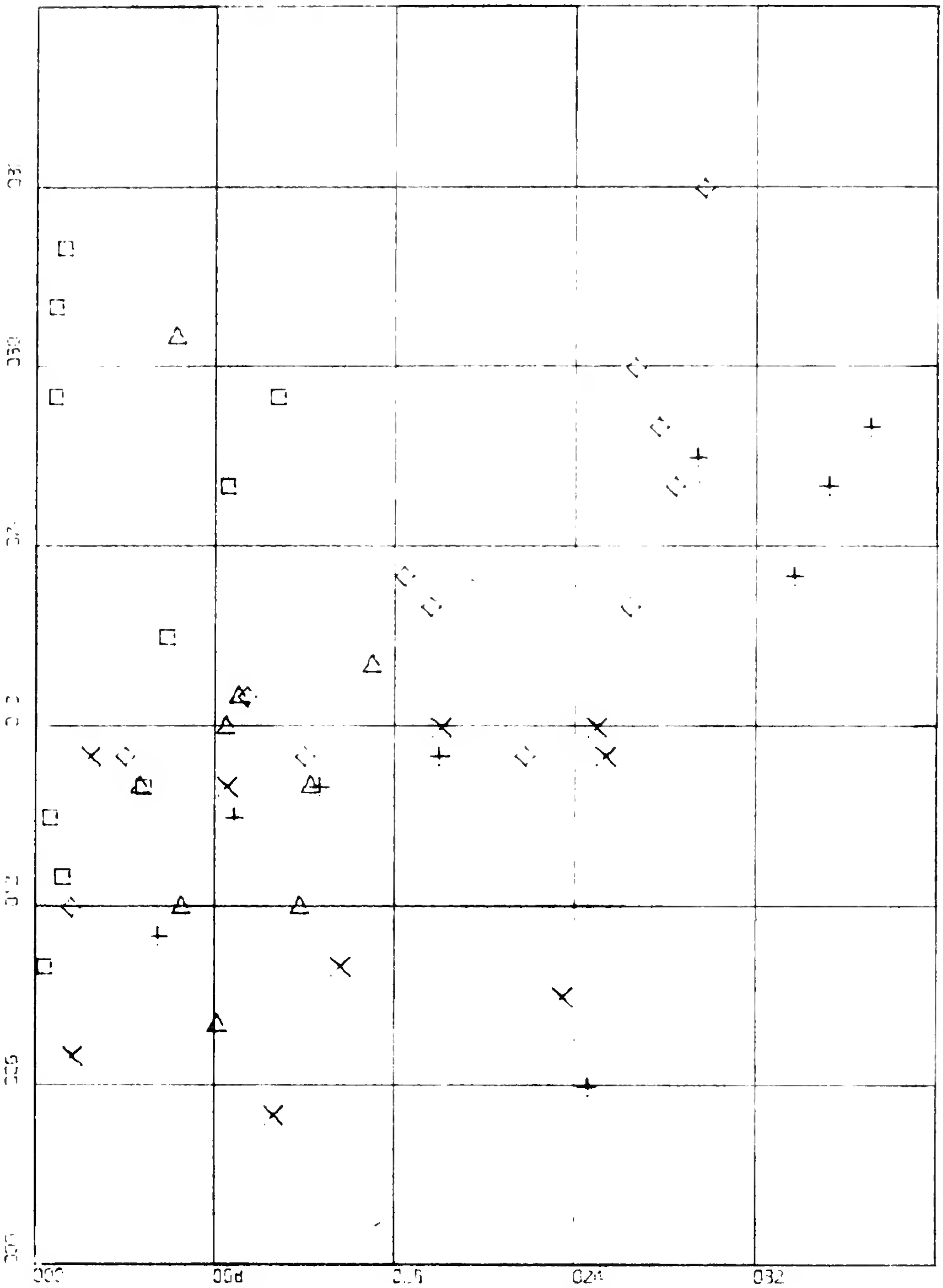
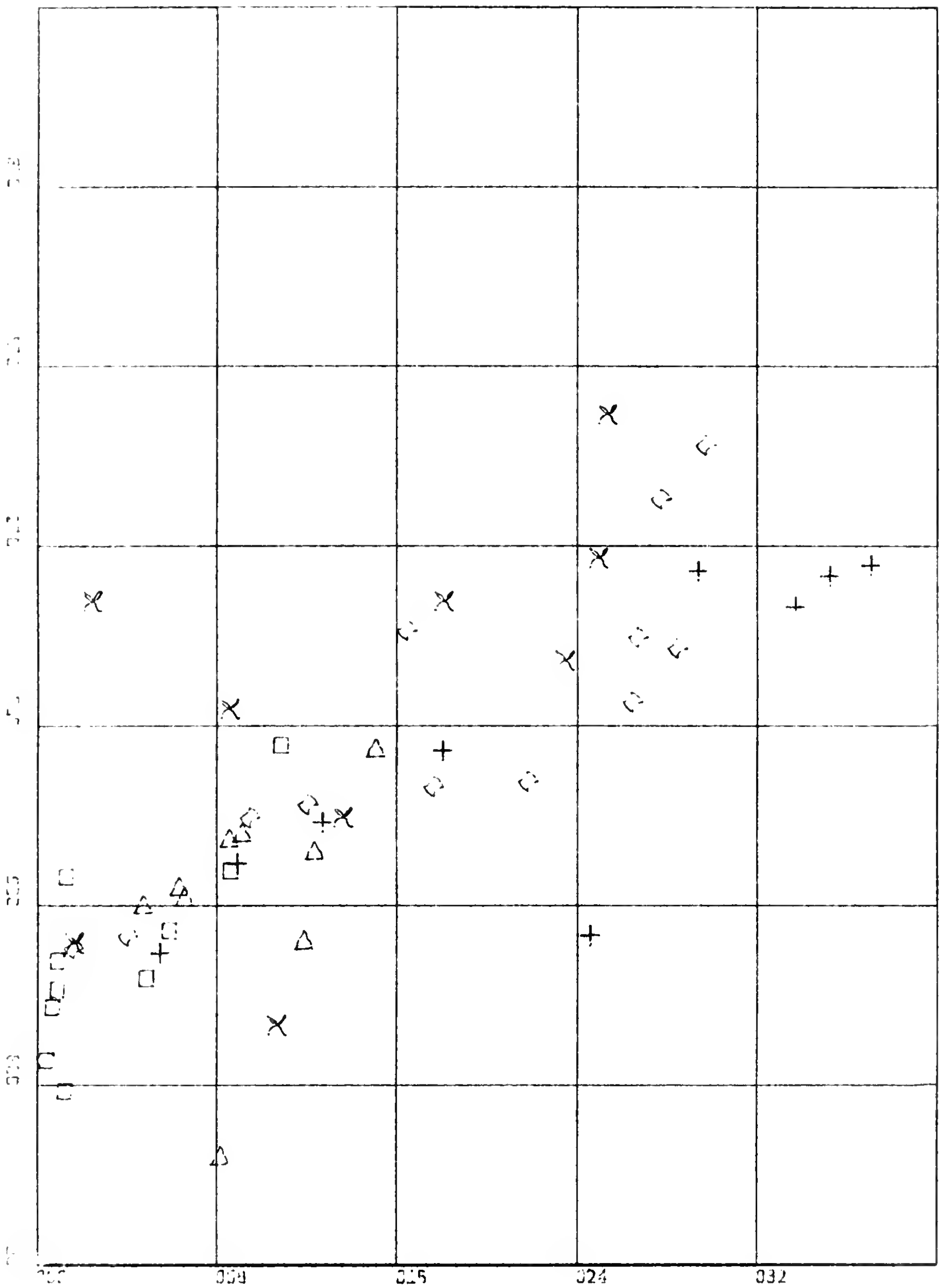


APPENDIX C

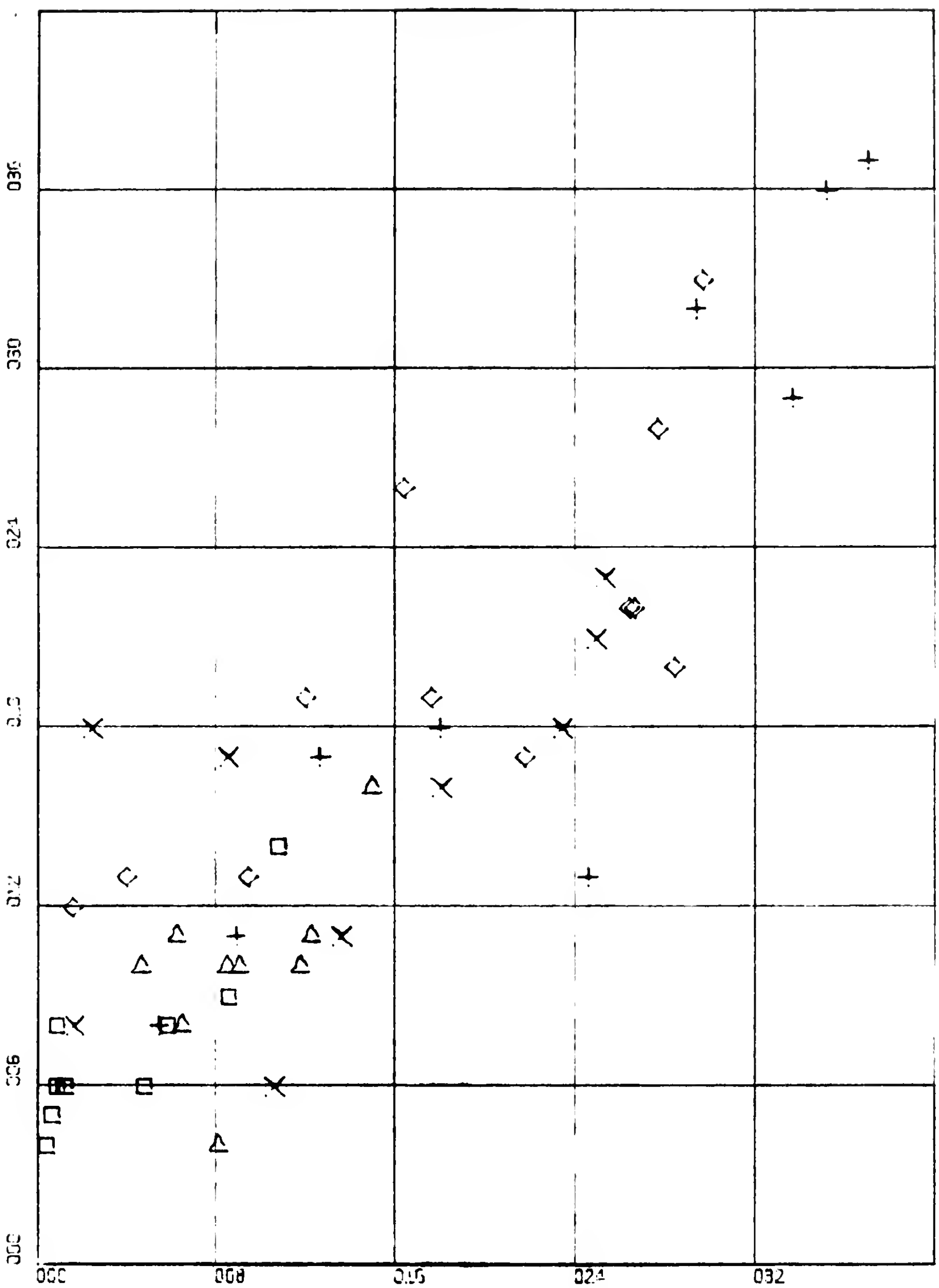
Plots of Hours Since Engine Oil Change
Versus Densities of Eight Elements for Five Engines



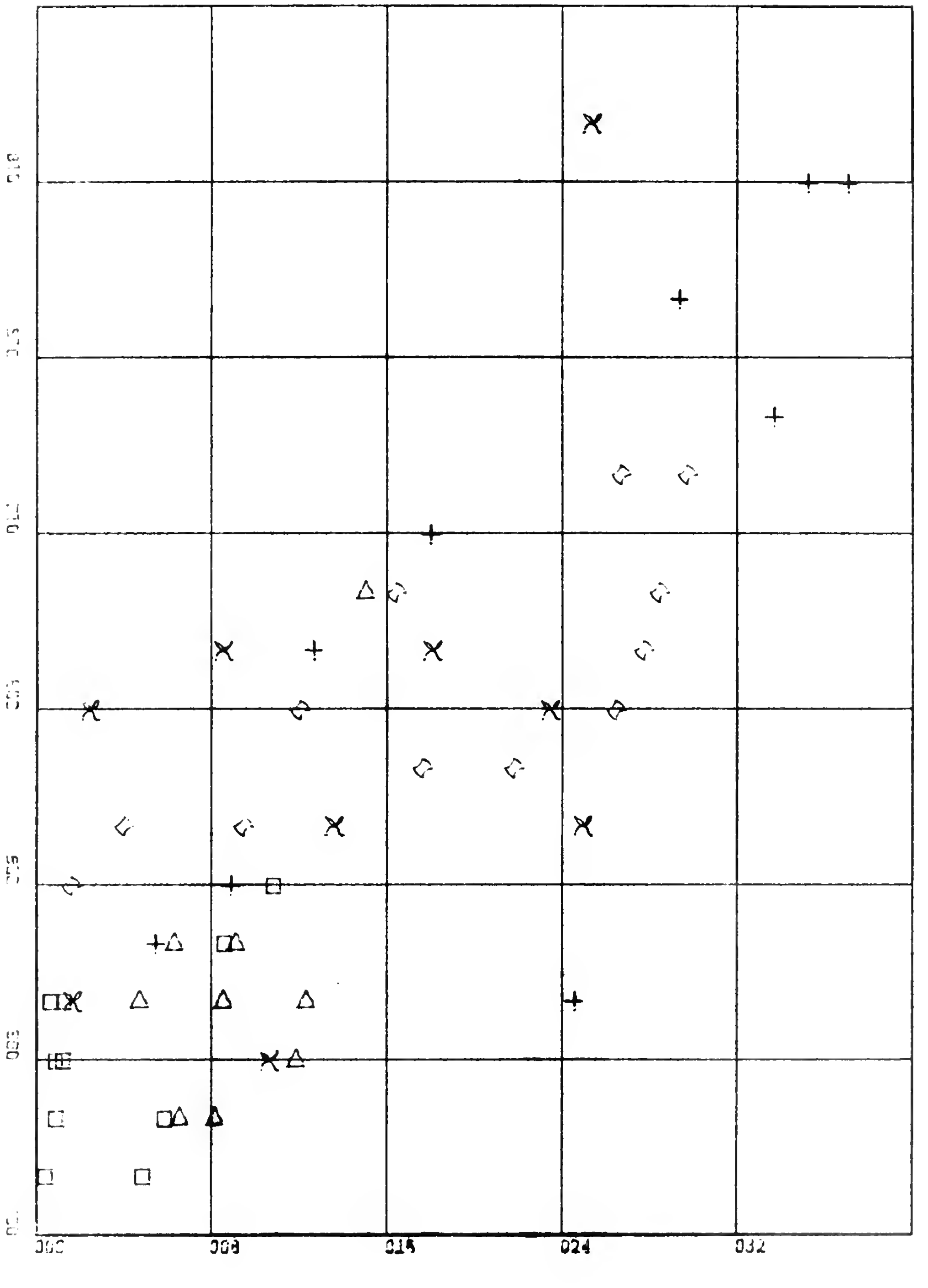
X-SCALE 8.00E+01 UNITS INCH.
Y-SCALE 6.00E+00 UNITS INCH.
OIL CHANGE VS. PPM OF ALUMINUM
CARTY MODEL NUMBER R20006



X-SCALE=8.00E+01 UNITS INCH.
Y-SCALE=3.00E+01 UNITS INCH.
OIL CHANGE VS. PPM OF IRON
CARTY MODEL NUMBER R20006

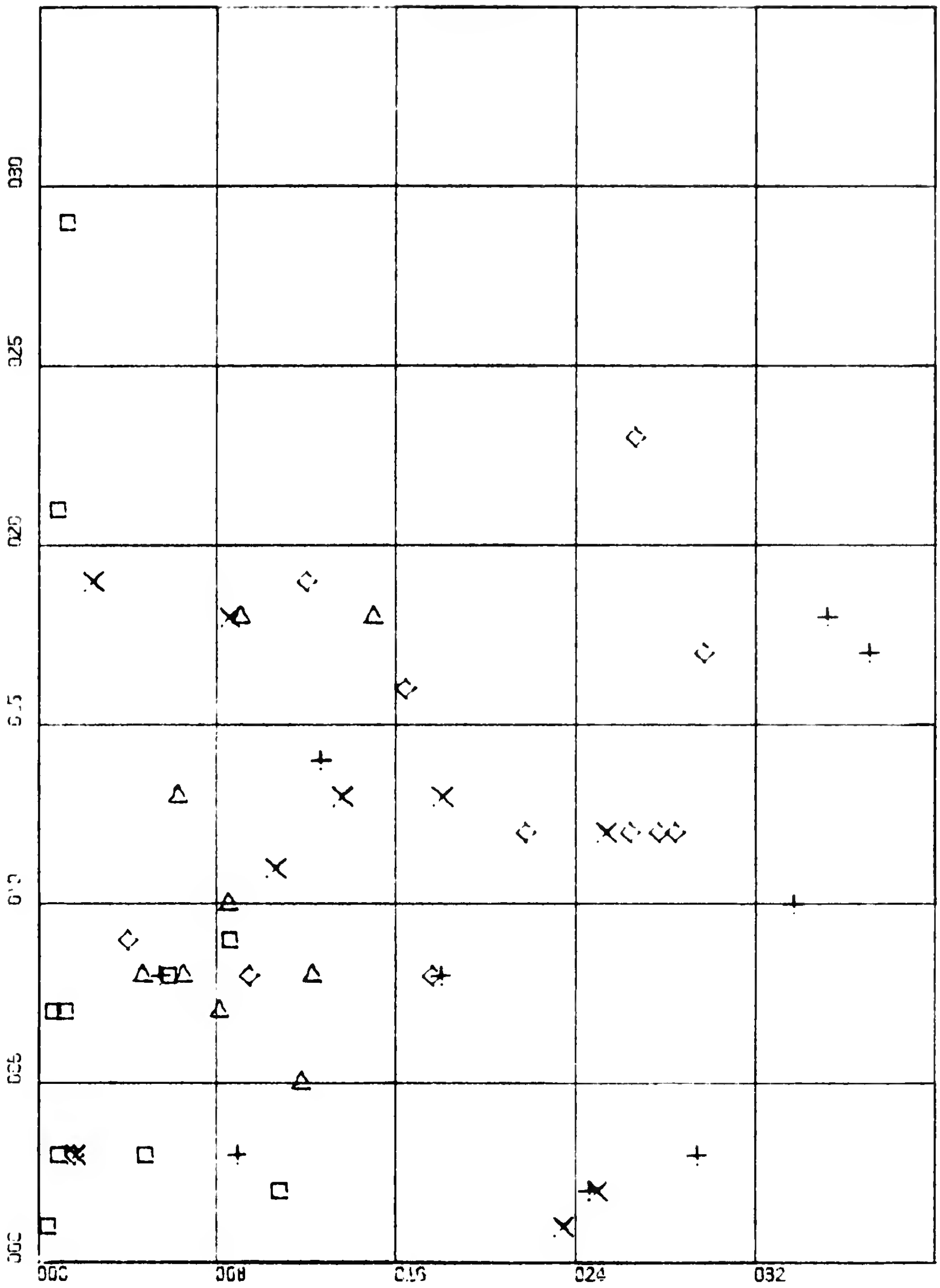


X-SCALE 8.00E+01 UNITS INCH.
Y-SCALE 6.00E+00 UNITS INCH.
OIL CHANGE VS. PPM OF COPPER
CARTY MODEL NUMBER R20006



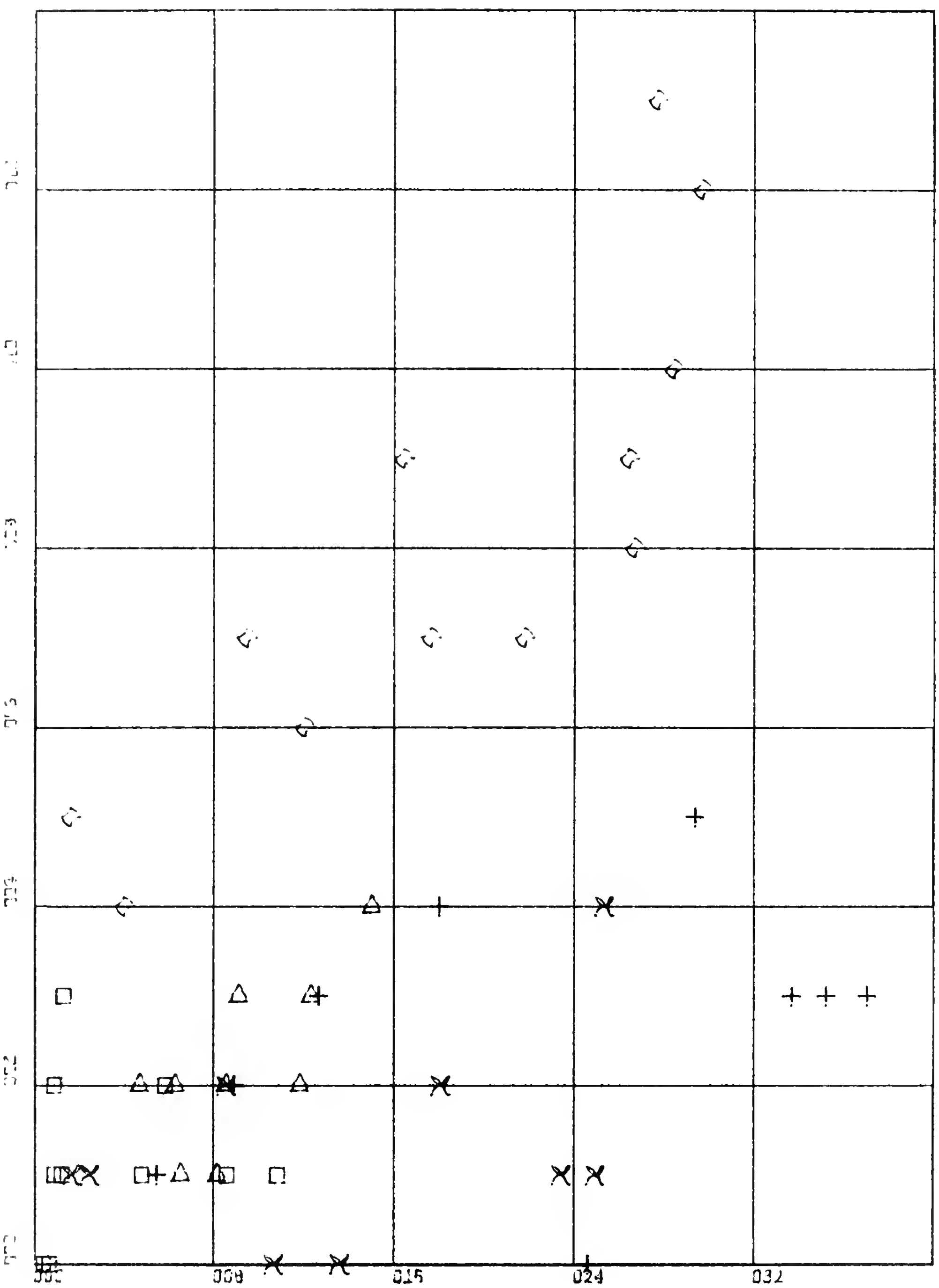
X-SCALE=8.00E+01 UNITS INCH.
Y-SCALE=3.00E+00 UNITS INCH.

OIL CHANGE VS. PPM OF SILVER
CARTY MODEL NUMBER R20006

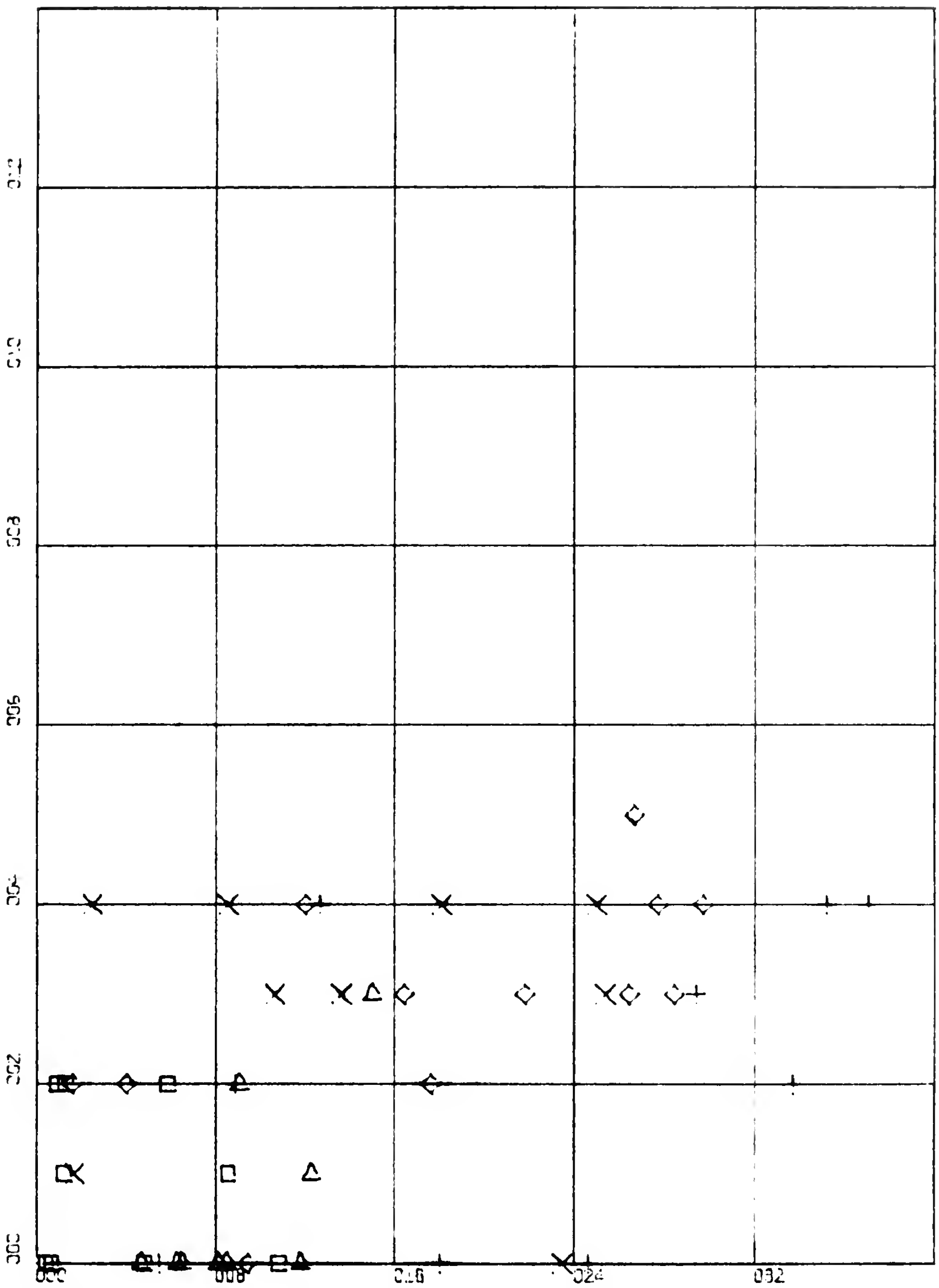


X-SCALE=8.00E+01 UNITS INCH.
Y-SCALE=5.00E+00 UNITS INCH.

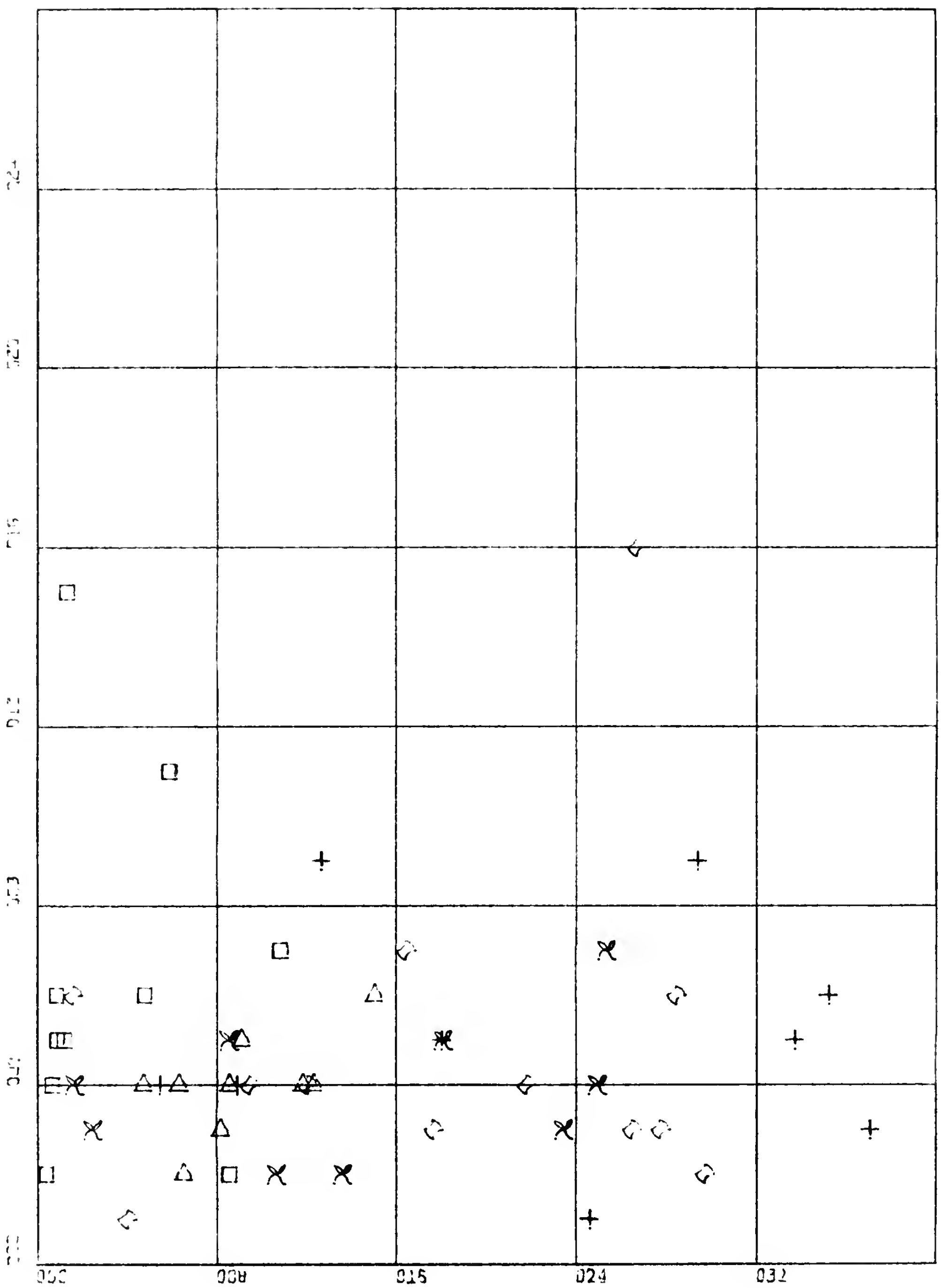
OIL CHANGE VS. PPM OF CHROMIUM
CARTY MODEL NUMBER R20006



X-SCALE=8.00E+01 UNITS INCH.
Y-SCALE=2.00E+00 UNITS INCH.
OIL CHANGE VS. PPM OF MAGNES.
CARTY MODEL NUMBER R20006



X-SCALE 8.00E+01 UNITS INCH.
Y-SCALE 2.00E+00 UNITS INCH.
OIL CHANGE VS. PPM OF NICKEL
CARTY MODEL NUMBER R20006



X-SCALE=8.00E+01 UNITS INCH.
Y-SCALE=4.00E+00 UNITS INCH.

OIL CHANGE VS. PPM OF SILICON
CARTY MODEL NUMBER R20006

APPENDIX D

STEPWISE REGRESSION RESULTS

First Independent Variable Entered in Stepwise Regression

Engine No.	Mg	Al	Fe	Cu	Ag	Cr	Ni	Si
1	2	2	2	1	2	2	2	2
2	1	2	2	2	1	2	2	1
3	2	2	2	2	2	1	1	2
4	2	2	2	2	2	2	1	2
5	2	2	2	2	2	1	2	1
6	1	1	1	2	2	2	1	2
7	1	2	2	2	2	2	1	1
8	2	2	1	2	2	1	1	2
9	2	2	2	2	2	1	1	1
10	1	1	2	2	2	2	2	2
11	2	2	1	2	1	2	1	2
12	1	2	2	2	2	1	1	1
13	2	2	2	2	2	1	2	1
14	1	1	1	1	1	2	1	2
15	1	1	1	1	1	1	2	2
16	2	2	2	2	2	2	2	1
17	2	1	2	2	2	1	2	2
18	2	2	2	2	2	1	2	2
19	2	2	2	2	2	1	1	1
20	2	1	2	2	2	2	2	2
21	1	1	2	2	2	1	2	2
22	1	2	2	2	2	2	1	2
23	1	1	2	2	2	1	1	2
24	2	2	2	2	2	1	1	2
25	1	1	1	2	1	1	1	1
26	2	2	2	2	1	1	2	2
Total 1	11	9	6	3	6	15	14	9
Total 2	15	17	20	23	20	11	12	17

Key: 1 Signifies that the independent variable, hours since engine overhaul, was the first variable entered in the stepwise regression.

2 Signifies that hours since engine oil change was the first variable entered in the stepwise regression.

APPENDIX E

TEST RESULTS FOR HYPOTHESES THAT THE REGRESSION LINE HAS ZERO SLOPE

Engine Number	F_E		C_U		A_G		A_L		C_R		M_G		N_I		S_I	
	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}
1	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
2	A	A	A	A	A	A	A	A	R	A	R	A	A	A	A	A
3	A	A	R	R	A	A	A	A	A	A	A	A	A	A	A	A
4	R	A	R	R	A	A	A	A	A	A	R	A	A	A	A	A
5	R	R	R	A	R	A	A	A	A	R	A	A	R	R	A	A
6	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
7	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
8	R	R	R	R	A	A	A	A	A	A	A	A	A	A	A	A
9	R	R	R	R	R	R	R	A	R	A	A	A	R	A	A	A
10	A	A	A	A	R	A	A	A	R	A	A	A	A	A	A	A
11	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
12	R	R	R	R	A	A	A	A	A	A	A	A	A	A	A	A
13	R	R	R	R	R	R	R	R	R	A	R	R	R	A	A	A
14	A	A	R	R	A	A	A	A	A	A	A	A	A	A	A	A
15	R	R	R	R	R	A	A	A	A	A	R	R	R	R	A	A
16	A	A	A	A	R	A	A	A	A	A	A	A	A	A	A	A
17	R	R	R	R	R	R	R	R	R	A	R	R	A	A	R	R
18	R	R	R	R	R	R	A	A	A	A	A	A	A	A	A	A
19	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
21	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
22	A	A	A	A	A	A	A	A	A	A	R	R	A	A	A	A
23	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A

Engine Number	F_E		C_U		A_G		A_L		C_R		M_G		N_I		S_I	
	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}
24	R	A	R	A	A	A	R	R	A	A	A	A	R	A	A	A
25	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
26	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
Total Reject	10	8	12	10	8	4	4	3	5	1	6	4	5	2	1	1
Total Accept	16	18	14	16	18	22	22	23	21	25	20	22	21	24	25	25

Key: A means accept the hypothesis.
B means reject the hypothesis.
All hypotheses tested at the = .05 level

H_{01} : $\beta_{1i} = 0$; $i = 1, 2, \ldots, 26$

H_{02} : $\beta_{2i} = 0$; $i = 1, 2, \ldots, 26$

for each element.

APPENDIX F

DATA FOR ALUMINUM

PREDICTION INTERVAL FOR AN INDIVIDUAL VALUEOF Y FOR A HYPOTHETICAL VALUE OF THE INDEPENDENT VARIABLE,X = 150

ENGINE NO.	PREDICTED LOW END	PREDICTED HIGH END
1	3.00000	27.00000
2	0.0	10.00000
3	8.00000	27.00000
4	0.0	19.00000
5	4.00000	32.00000
6	0.0	39.00000
7	1.00000	24.00000
8	3.00000	21.00000
9	2.00000	27.00000
10	2.00000	17.00000
11	4.00000	25.00000
12	3.00000	53.00000
13	11.00000	29.00000
14	8.00000	30.00000
15	0.0	33.00000
16	0.0	22.00000
17	14.00000	23.00000
18	6.00000	20.00000
19	3.00000	20.00000
20	3.00000	18.00000
21	2.00000	44.00000
22	0.0	19.00000
23	5.00000	28.00000
24	6.00000	13.00000
25	2.00000	32.00000
26	4.00000	11.00000

DATA FOR IRON

PREDICTION INTERVAL FOR AN INDIVIDUAL VALUEOF Y FOR A HYPOTHETICAL VALUE OF THE INDEPENDENT VARIABLE, X = 15C

ENGINE NO.	PREDICTED LOW END	PREDICTED HIGH END
1	49.00000	95.00000
2	25.00000	54.00000
3	47.00000	82.00000
4	45.00000	73.00000
5	59.00000	114.00000
6	19.00000	123.00000
7	38.00000	151.00000
8	91.00000	166.00000
9	40.00000	105.00000
10	38.00000	72.00000
11	47.00000	87.00000
12	62.00000	124.00000
13	56.00000	109.00000
14	32.00000	119.00000
15	30.00000	118.00000
16	17.00000	77.00000
17	79.00000	102.00000
18	38.00000	76.00000
19	44.00000	89.00000
20	44.00000	85.00000
21	42.00000	147.00000
22	44.00000	80.00000
23	35.00000	112.00000
24	30.00000	66.00000
25	26.00000	136.00000
26	29.00000	53.00000

DATA FOR COPPER

PREDICTICN INTERVAL FOR AN INDIVIDUAL VALUEOF Y FOR A HYPOTHETICAL VALUE OF THE INDEPENDENT VARIABLE,X = 150

ENGINE NO.	PREDICTED LOW END	PREDICTED HIGH END
1	6.00000	30.00000
2	4.00000	13.00000
3	10.00000	18.00000
4	7.00000	20.00000
5	10.00000	27.00000
6	0.0	22.00000
7	4.00000	26.00000
8	10.00000	25.00000
9	4.00000	26.00000
10	4.00000	14.00000
11	5.00000	19.00000
12	9.00000	19.00000
13	9.00000	27.00000
14	0.0	30.00000
15	5.00000	20.00000
16	3.00000	21.00000
17	18.00000	27.00000
18	8.00000	18.00000
19	0.0	23.00000
20	6.00000	20.00000
21	7.00000	23.00000
22	1.00000	19.00000
23	2.00000	13.00000
24	8.00000	22.00000
25	2.00000	32.00000
26	5.00000	17.00000

DATA FOR SILVER

PREDICTION INTERVAL FOR AN INDIVIDUAL VALUEOF Y FOR A HYPOTHETICAL VALUE OF THE INDEPENDENT VARIABLE, X = 150

ENGINE NO.	PREDICTED LOW END	PREDICTED HIGH END
1	0.0	12.00000
2	0.0	5.00000
3	4.00000	14.00000
4	1.00000	12.00000
5	3.00000	16.00000
6	0.0	18.00000
7	0.0	18.00000
8	0.0	11.00000
9	0.0	16.00000
10	1.00000	9.00000
11	1.00000	8.00000
12	0.0	10.00000
13	4.00000	12.00000
14	1.00000	16.00000
15	0.0	13.00000
16	0.0	11.00000
17	3.00000	14.00000
18	4.00000	14.00000
19	0.0	13.00000
20	1.00000	13.00000
21	1.00000	11.00000
22	0.0	9.00000
23	0.0	7.00000
24	0.0	10.00000
25	0.0	18.00000
26	0.0	8.00000

DATA FOR CHROMIUM

PREDICTION INTERVAL FOR AN INDIVIDUAL VALUEOF Y FOR A HYPOTHETICAL VALUE OF THE INDEPENDENT VARIABLE, X = 150

ENGINE NO.	PREDICTED LOW END	PREDICTED HIGH END
1	0.0	19.00000
2	0.0	12.00000
3	0.0	16.00000
4	0.0	16.00000
5	0.0	18.00000
6	1.00000	21.00000
7	0.0	24.00000
8	0.0	23.00000
9	0.0	20.00000
10	0.0	18.00000
11	0.0	39.00000
12	0.0	27.00000
13	1.00000	21.00000
14	0.0	25.00000
15	0.0	25.00000
16	0.0	17.00000
17	1.00000	25.00000
18	0.0	13.00000
19	0.0	16.00000
20	0.0	14.00000
21	2.00000	30.00000
22	0.0	13.00000
23	0.0	23.00000
24	0.0	16.00000
25	0.0	18.00000
26	0.0	15.00000

DATA FOR MAGNESIUM

PREDICTION INTERVAL FOR AN INDIVIDUAL VALUEOF Y FOR A HYPOTHETICAL VALUE OF THE INDEPENDENT VARIABLE, X = 150

ENGINE NO.	PREDICTED LOW END	PREDICTED HIGH END
1	0.0	4.00000
2	0.0	2.00000
3	2.00000	6.00000
4	0.0	3.00000
5	0.0	6.00000
6	0.0	7.00000
7	0.0	4.00000
8	0.0	6.00000
9	0.0	6.00000
10	0.0	2.00000
11	0.0	4.00000
12	0.0	4.00000
13	3.00000	11.00000
14	0.0	6.00000
15	0.0	6.00000
16	0.0	3.00000
17	1.00000	5.00000
18	0.0	4.00000
19	0.0	3.00000
20	0.0	3.00000
21	0.0	5.00000
22	0.0	4.00000
23	0.0	6.00000
24	0.0	3.00000
25	0.0	7.00000
26	0.0	3.00000

DATA FOR NICKEL

PREDICTION INTERVAL FOR AN INDIVIDUAL VALUEOF Y FOR A HYPOTHETICAL VALUE OF THE INDEPENDENT VARIABLE, X = 150

ENGINE NO.	PREDICTED LOW END	PREDICTED HIGH END
1	0.0	7.00000
2	0.0	3.00000
3	0.0	4.00000
4	0.0	4.00000
5	0.0	5.00000
6	0.0	5.00000
7	0.0	6.00000
8	0.0	5.00000
9	0.0	5.00000
10	0.0	6.00000
11	0.0	4.00000
12	0.0	3.00000
13	0.0	5.00000
14	0.0	5.00000
15	0.0	5.00000
16	0.0	4.00000
17	0.0	3.00000
18	0.0	3.00000
19	0.0	3.00000
20	0.0	5.00000
21	0.0	6.00000
22	0.0	4.00000
23	0.0	5.00000
24	0.0	4.00000
25	0.0	8.00000
26	0.0	3.00000

DATA FOR SILICON

PREDICTION INTERVAL FOR AN INDIVIDUAL VALUEOF Y FOR A HYPOTHETICAL VALUE OF THE INDEPENDENT VARIABLE, X = 150

ENGINE NO.	PREDICTED LOW END	PREDICTED HIGH END
1	0.0	10.00000
2	0.0	7.00000
3	1.00000	8.00000
4	0.0	6.00000
5	0.0	31.00000
6	0.0	10.00000
7	0.0	8.00000
8	0.0	9.00000
9	0.0	11.00000
10	0.0	22.00000
11	0.0	8.00000
12	0.0	18.00000
13	0.0	13.00000
14	1.00000	7.00000
15	1.00000	8.00000
16	0.0	7.00000
17	1.00000	6.00000
18	0.0	7.00000
19	0.0	10.00000
20	0.0	7.00000
21	1.00000	8.00000
22	0.0	7.00000
23	0.0	9.00000
24	0.0	8.00000
25	0.0	9.00000
26	0.0	7.00000

APPENDIX G

Computer Program

```
C
C
C     THIS PROGRAM TAKES THE INDEPENDENT VARIABLE,OIL CHANGE
C     AND REGRESSES IT AGAINST THE DEPENDENT VARIABLE, PPM
C
C
      REAL *8 AMOD/'R20006  '/,SN(27),D,A,STORE(10)/10*0.0/
      REAL *4 HOV(27,15),HOC(27,15),AL(27,15),FE(27,15),CR(2
     17,15),AG(27,15),CU(27,15),TN(27,15),MG(27,15),TI(27,15
     2),NI(27,15),SI(27,15),DATE(27,15)
      INTEGER * 2  I(27)/27 *0/
      DATA  HOV,HOC,AL,FE,CR,AG,CU,TN,MG,TI,NI,SI,DATE /5265
     1*0.0/,SN/'101196  ','103105  ','104166  ','105005  ','
     2105246  ','106505  ','106666  ','106681  ','106690  ',
     3'106707  ','106797  ','106802  ','107486  ','107935  '
     4,'108003  ','108372  ','108458  ','108461  ','108553
     5','108616  ','108919  ','700002  ','700136  ','700163
     6 ','702269  ','702489  ','105189  '/
      DO 1 IX=1,241
      READ(5,7) A,D,E2,E3,E4,E5,E6,E7,E8,E9,E10,E11,E12,E13,
     1E14
    7 FORMAT(1X,A8,1X,A8,14X,A4,2X,F4.0,F3.0,5X,10F3.0)
      DO 14 L=1,26
      IF ( D .EQ. SN(L)) GO TO 31
   14 CONTINUE
      GO TO 1
   31 I(L)=I(L)+1
      LA = I(L)
      DATE(L,LA) = E2
      HOV(L,LA) = E3
      HOC(L,LA) = E4
      AL(L,LA) = E5
      FE(L,LA) = E6
      CR(L,LA) = E7
      AG(L,LA) = E8
      CU(L,LA) = E9
      MG(L,LA) = E11
      NI(L,LA) = E13
      SI(L,LA)=E14
    1 CONTINUE
      WRITE(6,200)
  200 FORMAT('1',///57X,'DATA FOR ALUMINUM')
      CALL REGRES(HOC,AL,I,1)
      WRITE(6,201)
  201 FORMAT('1',///57X,'DATA FOR IRON')
      CALL REGRES(HOC,FE,I,2)
      WRITE(6,202)
  202 FORMAT('1',///57X,'DATA FOR CHROMIUM')
      CALL REGRES(HOC,CR,I,3)
      WRITE(6,203)
  203 FORMAT('1',///57X,'DATA FOR SILVER')
      CALL REGRES(HOC,AG,I,4)
      WRITE(6,206)
  206 FORMAT('1',///57X,'DATA FOR MAGNESIUM')
      CALL REGRES(HOC,MG,I,5)
      WRITE(6,205)
  205 FORMAT('1',///57X,'DATA FOR NICKEL')
      CALL REGRES(HOC,NI,I,6)
      WRITE(6,204)
  204 FORMAT('1',///57X,'DATA FOR COPPER')
      CALL REGRES(HOC,CU,I,7)
      WRITE(6,207)
  207 FORMAT('1',///57X,'DATA FOR SILICON')
      CALL REGRES(HOC,SI,I,8)
      STOP
      END
```

```
      SUBROUTINE REGRES(X,Y,I,NUMEL)
C
C
C        THIS SUBPROGRAM USES X AS THE  INDEPENDENT VARIABLE
C        AND Y IS THE DEPENDENT VARIABLE. THE I IN THE
C        ARGUMENT IS THE NUMBER OF OBSERVATIONS ON EACH
C        SERIALLY NUMBERED R20006 ENGINE
C
C
      DIMENSION X(27,17),Y(27,17),I(27),ALFA(27),XMEAN(27)
     1,CAPY(27,17),SSQR(27),YMEAN(27),BETA(27)
      DIMENSION RHC(27),SSQRX(27)
      DIMENSION VARX(27),YHATHI(27),YHATLO(27)
      INTEGER*2 I
      DATA T6/1.943/,T7/1.895/,T8/1.860/,T9/1.833/,
     1T11/1.796/,T12/1.782/,T10/1.812/
C     COMPUTE YMEAN FOR FUTURE USE
      DO 1 J =1,26
      SUMY = 0.0
      K = I(J)
      DO 2 L = 1,K
    2 SUMY = SUMY + Y(J,L)
    1 YMEAN(J) = SUMY/I(J)
C     COMPUTE BETA FOR Y = ALFA + BETA*X FOR EACH OF THE 26
C     ENGINES
      DO 3 M = 1,26
      SUMX = 0.0
      N=I(M)
      DO 4 NN = 1,N
    4 SUMX = SUMX + X(M,NN)
    3 XMEAN(M) = SUMX / I(M)
      DO 5 KK = 1,26
      SUMP = 0.0
      SSX  = 0.0
      KI = I(KK)
      DO 6 LL = 1,KI
      SUMP = SUMP + ((X(KK,LL)-XMEAN(KK))*(Y(KK,LL)-YMEAN(KK
     1)))
    6 SSX = SSX + (X(KK,LL)-XMEAN(KK))**2
      SSQRX(KK) = SSX/(I(KK) - 1)
    5 BETA(KK) = SUMP/ SSX
C     COMPUTE ALFA FOR  Y = ALFA + BETA*X FOR EACH OF THE 26
C     ENGINES
      DO 12 IL=1,26
   12 ALFA(IL) = YMEAN(IL) - BETA(IL)*XMEAN(IL)
      DO 7  JX = 1,26
      KX = I(JX)
      DO 7  KXX = 1,KX
    7 CAPY(JX,KXX) =  ALFA(JX) + BETA(JX)*X(JX,KXX)
C     NOW COMPUTE THE ESTIMATE OF THE CONDITIONAL VARIANCE
C     OF Y GIVEN X FOR EACH OF THE 26 ENGINES
      DO 8  MQ = 1,26
      NQ = I(MQ)
      SAM = 0.0
      DO 9  NNQ = 1,NQ
    9 SAM = SAM +(Y(MQ,NNQ)-CAPY(MQ,NNQ))**2
    8 SSQR(MQ) = SAM/(NQ-2)
C
C     COMPUTE THE CORRELATION COEFFICIENT BETWEEN X AND Y,
C     AND COMPUTE THE VARIANCE OF X
C     SEE P. 158, NAVORD REPORT 3369
C
```

```
      DO 16 NZ = 1,26
      XY = 0.0
      EX = 0.0
      WYE = 0.0
      EXSQR = 0.0
      WYESQR = 0.0
      NZD = I(NZ)
      DO 15 NZZ =1,NZD
      XY = XY + X(NZ,NZZ)*Y(NZ,NZZ)
      EX = EX + X(NZ,NZZ)
      WYE = WYE + Y(NZ,NZZ)
      EXSQR = EXSQR + X(NZ,NZZ)**2
   15 WYESQR = WYESQR + Y(NZ,NZZ)**2
      VARX(NZ) = ((I(NZ)*EXSQR)-EX**2)/(I(NZ)*(I(NZ)-1))
   16 RHO(NZ) = (NZD * XY - EX * WYE)/SQRT((NZD*EXSQR -
     1 EX**2)*(NZD*WYESQR - WYE**2))
      WRITE(6,100) (NO,ALFA(NO),BETA(NO),SSQR(NO),RHO(NO),
     1NO=1,26)
  100 FORMAT(////30X,'ENGINE NO.',10X,'ALFA',10X,'BETA',10X
     1,'VARIANCE',10X,'RHO',////(35X,I2,8X,F10.5,5X,F10.5,
     25X,F10.5,5X,F10.5))
      CALL BARTLT(SSQR,I)
C
C
C     NOW WE ESTABLISH A 95 PER CENT PREDICTION INTERVAL FOR
C     Y FOR EACH OF THE 26 ENGINES, CONSIDERING A HYPOTHET-
C     ICAL X-READING OF X = 150
C     FOR EACH ENGINE. SEE P. 163,NAVORD REPORT 3369
C
C
      DO 18 IX = 1,26
      RAD = SQRT(1.0 + 1.0/I(IX) +((150.0 - XMEAN(IX))**2)/(
     1(I(IX)-1)*VARX(IX)))
      IF( I(IX) .EQ. 8) GO TO 20
      IF( I(IX) .EQ. 9) GO TO 21
      IF( I(IX) .EQ.10) GO TO 22
      IF( I(IX) .EQ.11) GO TO 23
      IF( I(IX) .EQ.12) GO TO 24
      IF( I(IX) .EQ.13) GO TO 25
      IF( I(IX) .EQ.14) GO TO 26
   20 T=T6
      GO TO 30
   21 T=T7
      GO TO 30
   22 T=T8
      GO TO 30
   23 T=T9
      GO TO 30
   24 T=T10
      GO TO 30
   25 T=T11
      GO TO 30
   26 T=T12
   30 YHAT = ALFA(IX) + BETA(IX) * 150.0
      YHATHI(IX) = YHAT + T* SQRT(SSQR(IX))* RAD
      YHATLO(IX) = YHAT - T* SQRT(SSQR(IX))* RAD
      IF(YHATHI(IX) .LE. 0.0) GO TO 50
      ITRUNC = YHATHI(IX)
      YHATHI(IX) = ITRUNC + 1
      GO TO 52
   50 YHATHI(IX) = 0.0
   52 IF(YHATLO(IX) .LE. 0.0) GO TO 51
      JTRUNC = YHATLO(IX)
      YHATLO(IX) = JTRUNC - 1
      IF(YHATLO(IX) .LT. 0.0) YHATLO(IX) = 0.0
      GO TO 18
   51 YHATLO(IX) = 0.0
   18 CONTINUE
      IF(NUMEL .EQ. 1) GO TO 301
      IF(NUMEL .EQ. 2) GO TO 302
      IF(NUMEL .EQ. 3) GO TO 303
      IF(NUMEL .EQ. 4) GO TO 304
```

```
      IF(NUMEL .EQ. 5) GO TO 305
      IF(NUMEL .EQ. 6) GO TO 306
      IF(NUMEL .EQ. 7) GO TO 307
      IF(NUMEL .EQ. 8) GO TO 308
  301 WRITE(6,401)
  401 FORMAT('1',///////57X,'DATA FOR ALUMINUM')
      GO TO 199
  302 WRITE(6,402)
  402 FORMAT('1',///////57X,'DATA FOR IRON')
      GO TO 199
  303 WRITE(6,403)
  403 FORMAT('1',///////57X,'DATA FOR CHROMIUM')
      GO TO 199
  304 WRITE(6,404)
  404 FORMAT('1',///////57X,'DATA FOR SILVER')
      GO TO 199
  305 WRITE(6,405)
  405 FORMAT('1',///////57X,'DATA FOR MAGNESIUM')
      GO TO 199
  306 WRITE(6,406)
  406 FORMAT('1',///////57X,'DATA FOR NICKEL')
      GO TO 199
  307 WRITE(6,407)
  407 FORMAT('1',///////57X,'DATA FOR COPPER')
      GO TO 199
  308 WRITE(6,408)
  408 FORMAT('1',///////57X,'DATA FOR SILICON')
  199 WRITE(6,200)(NR,YHATLO(NR),YHATHI(NR),NR=1,26)
  200 FORMAT(     ////39X,'PREDICTION INTERVAL FOR AN ',
     1INDIVIDUAL VALUEOF Y FOR A',//39X,'HYPOTHETICAL VALUE
     2OF THE INDEPENDENT VARIABLE,X = 150',////63X,
     3'PREDICTED',13X,'PREDICTED',/40X,'ENGINE NO.',14X,'LOW
     4END',15X,'HIGH END',///(44X,I2,17X,F9.5,13X,F9.5))
      RETURN
      END


      SUBROUTINE BARTLT(S,I)
C
C
C      THIS SUBPROGRAM COMPUTES BARTLETTS STATISTIC FOR
C     TESTING THE HOMOGENEITY OF VARIANCES OF A SET OF K
C     OBSERVATIONS. THEARGUMENTS ARE----S IS THE SET OF
C     ESTIMATED VARIANCES FOR EACH OF THE SETS OF DATA, AND
C     I IS THE NUMBER OF OBSERVATIONS WITHIN EACH SET
C     OF DATA.
C
C
      DIMENSION S(27),I(27),DF(27)
      INTEGER*2 I

C     COMPUTE THE DEGREES OF FREEDOM FOR EACH SSQR

      DO 1 J = 1,26
    1 DF(J)=I(J)-2

C     COMPUTE THE TOTAL DEGREES OF FREEDOM- REF. PAGE 225,
C     BROWNLEE

      F = 0.0
      DO 2 K = 1,26
    2 F = F + DF(K)

C     COMPUTE SAMPLE ESTIMATE OF SIGMA SQUARE

      SUM = 0.0
      DO 3 L = 1,26
    3 SUM = SUM + (DF(L) * S(L))
      SIGSQR = SUM/F
      B = 0.0
      DO 4 M= 1,26
      IF(S(M) .LE. 0.0) GO TO 4
```

```
      B = B+ DF(M) * ALOG(S(M)/SIGSQR)
    4 CONTINUE
      B = (-B)
      FLIPF = C.0
      DO 5 N = 1,26
    5 FLIPF = FLIPF + (1.C/DF(N)-1.C/F)
      C = 1.0 + (1.0/75.0)*FLIPF
      STAT = B/C
C
C     BARTLETTS STATISTIC IS DISTRIBUTED APPROXIMATELY AS
C     CHI SQUARE WITH 25 DEGREES OF FREEDOM
C
      WRITE(6,300) STAT
  300 FORMAT(/////53X,'BARTLETTS STATISTIC',//57X,F10.2)
      WRITE(6,301)
  301 FORMAT(/////50X,'CHI SQUARE STATISTIC AT .05 LEVEL',//
     163X,'37.7')
      IF(STAT .GT.   37.7) GO TO 302
      WRITE(6,303)
  303 FORMAT(///40X,'ACCEPT THE HYPOTHESIS OF HOMOGENEITY OF
     1 VARIANCES')
      GO TO 6
  302 WRITE(6,304)
  304 FORMAT(///40X,'REJECT THE HYPOTHESIS OF HOMOGENEITY OF
     1 VARIANCES')
    6 RETURN
      END
```

DATA FOR ALUMINUM

ENGINE NO.	ALFA	BETA	VARIANCE	RHO
1	13.30434	0.01470	28.47899	0.19321
2	6.84295	-0.00841	3.06688	-0.37964
3	16.97585	0.00576	16.77626	0.11472
4	9.42757	0.00528	16.31842	0.12639
5	15.16684	0.02224	39.99223	0.33111
6	13.33934	0.00746	97.84805	0.07515
7	10.42277	0.01797	27.71808	0.31995
8	11.34949	0.00764	14.41651	0.13060
9	8.34820	0.04577	33.21201	0.71203
10	9.85517	0.00189	9.81777	0.04885
11	11.59952	0.02414	20.62318	0.35753
12	20.61287	0.05316	78.79721	0.23280
13	12.08619	0.05589	15.97491	0.82293
14	18.33127	0.007[illegible]3	31.08417	0.17799
15	18.17804	-0.01410	48.73520	-0.07127
16	15.07593	-0.03457	30.71198	-0.58394
17	6.42319	0.08271	3.16762	0.94023
18	11.14837	0.01539	10.06879	0.50956
19	9.73746	0.01489	15.26734	0.40384
20	10.34900	0.00272	12.40499	0.08329
21	18.35741	0.03504	88.08536	0.31535
22	6.34119	0.02662	17.39737	0.41019
23	15.30438	0.01323	24.53433	0.16335
24	6.53753	0.02333	1.68822	0.89707
25	18.72218	-0.00968	49.54111	-0.15742
26	7.40204	0.00465	1.60781	0.33606

BARTLETTS STATISTIC

76.73

CHI SQUARE STATISTIC AT .05 LEVEL

37.7

REJECT THE HYPOTHESIS OF HOMOGENEITY OF VARIANCES

DATA FOR IRON

ENGINE NO.	ALFA	BETA	VARIANCE	RHO
1	55.20422	0.11448	114.27705	0.60787
2	43.46326	-0.02378	44.85501	-0.29018
3	52.85899	0.07885	62.23904	0.63434
4	49.46271	0.06691	42.35352	0.70755
5	57.84926	0.19398	157.84903	0.83873
6	65.88390	0.03499	693.29321	0.13161
7	63.63976	0.21088	765.40845	0.60084
8	175.39531	-0.30968	314.39453	-0.75280
9	44.46706	0.19103	242.69214	0.84279
10	46.93655	0.05693	65.14589	0.49748
11	59.78769	0.05300	86.07813	0.38045
12	39.87607	0.35560	121.76297	0.78992
13	48.83186	0.22904	184.28542	0.86796
14	77.23982	-0.00702	521.75610	-0.04170
15	41.75443	0.21654	342.75439	0.38240
16	63.06525	-0.10498	205.27632	-0.64542
17	45.61081	0.30287	24.24014	0.96454
18	39.40042	0.12059	88.71730	0.84245
19	57.02637	0.06641	129.07980	0.56075
20	61.74995	0.02155	99.17853	0.22816
21	79.81621	0.10270	597.93677	0.35013
22	53.77707	0.05946	69.42007	0.44931
23	55.58455	0.12508	311.30054	0.41282
24	36.29274	0.08333	66.22913	0.75678
25	84.57494	-0.01996	731.59912	-0.08520
26	37.28738	0.02948	29.57013	0.46688

BARTLETTS STATISTIC

81.83

CHI SQUARE STATISTIC AT .05 LEVEL

37.7

REJECT THE HYPOTHESIS OF HOMOGENEITY OF VARIANCES

DATA FOR COPPER

ENGINE NO.	ALFA	BETA	VARIANCE	RHO
1	16.10133	0.01517	28.13663	0.20029
2	7.29360	0.01121	3.33279	0.46456
3	5.36026	0.06126	2.28625	0.95766
4	7.51995	0.04244	7.06022	0.84119
5	11.68334	0.04901	13.07191	0.80406
6	8.48832	0.01084	31.99890	0.18802
7	9.83609	0.03832	24.00394	0.61076
8	8.07563	0.06672	10.72700	0.80012
9	2.78638	0.08577	24.03470	0.91274
10	7.53345	0.01489	3.99420	0.51817
11	9.31796	0.02058	7.38620	0.47882
12	4.93100	0.06392	2.42263	0.85400
13	11.09875	0.04996	18.08391	0.77262
14	16.83813	-0.00636	57.65799	-0.11307
15	5.09610	0.05363	7.54217	0.56845
16	14.84856	-0.01592	14.05688	-0.43984
17	3.32089	0.13100	3.68555	0.97090
18	9.26964	0.02552	4.68321	0.82145
19	8.68514	0.02103	28.38513	0.41594
20	11.42535	0.01353	11.07911	0.40281
21	12.43534	0.02057	11.64888	0.47266
22	7.85786	0.01671	15.25947	0.28859
23	5.55491	0.01552	5.26603	0.39678
24	9.97796	0.03737	9.29325	0.81096
25	16.07785	0.01059	50.29272	0.17058
26	7.91086	0.02306	5.28834	0.69873

BARTLETTS STATISTIC

72.41

CHI SQUARE STATISTIC AT .05 LEVEL

37.7

REJECT THE HYPOTHESIS OF HOMOGENEITY OF VARIANCES

DATA FOR SILVER

ENGINE NO.	ALFA	BETA	VARIANCE	RHO
1	3.22107	0.01821	7.22279	0.43585
2	0.91387	0.00548	1.42715	0.36451
3	8.30336	0.00955	4.05259	0.36275
4	6.01645	0.00471	4.70592	0.20703
5	6.18161	0.02642	6.30180	0.72415
6	7.14782	0.00980	20.69699	0.21049
7	5.00347	0.02554	17.94392	0.51101
8	2.44647	0.02110	6.06984	0.48917
9	3.08596	0.03668	12.66748	0.79619
10	3.12557	0.01790	2.48596	0.67819
11	6.60215	-0.00961	1.52119	-0.48929
12	2.24322	0.02413	2.18181	0.54682
13	5.92650	0.01843	2.22144	0.78825
14	9.48374	-0.00355	10.91075	-0.14432
15	-0.02355	0.04886	5.32845	0.59947
16	8.42144	-0.02204	7.28558	-0.68556
17	-0.56630	0.06101	4.77864	0.85632
18	6.49206	0.01983	3.50315	0.79120
19	2.57543	0.02249	12.42607	0.59432
20	7.36988	0.00182	7.18613	0.07334
21	3.94133	0.01677	4.41735	0.57907
22	1.12335	0.01359	6.06122	0.36266
23	1.22760	0.00936	3.55870	0.30231
24	3.38863	0.01132	3.61304	0.55839
25	8.50132	0.00136	18.67505	0.03652
26	2.92798	-0.00034	4.14473	-0.01628

BARTLETTS STATISTIC

47.60

CHI SQUARE STATISTIC AT .05 LEVEL

37.7

REJECT THE HYPOTHESIS OF HOMOGENEITY OF VARIANCES

DATA FOR CHROMIUM

ENGINE NO.	ALFA	BETA	VARIANCE	RHO
1	-2.44742	0.06730	25.43413	0.69028
2	-0.97095	0.04212	9.39050	0.76127
3	13.80384	-0.04601	17.70932	-0.66800
4	7.70363	-0.01336	24.95369	-0.25200
5	7.61532	0.00654	17.21991	0.15536
6	14.70743	-0.01917	21.98987	-0.37804
7	14.52510	-0.03000	42.90073	-0.41159
8	6.42787	-0.00533	63.19046	-0.04386
9	4.43603	0.02100	34.74403	0.41414
10	-0.54279	0.06128	18.25578	0.75910
11	7.92736	0.04316	136.48221	0.25714
12	11.38564	-0.06720	85.03409	-0.27968
13	6.93963	0.03053	22.88821	0.55148
14	12.97217	-0.00834	51.34369	-0.15603
15	6.83300	0.04071	23.67117	0.28389
16	10.59030	-0.02582	22.68594	-0.53017
17	2.40704	0.07504	28.04414	0.64400
18	2.73783	0.01266	16.90269	0.35190
19	4.06411	0.01100	25.95224	0.24264
20	3.93629	0.00669	21.74173	0.15354
21	12.25228	0.02987	38.51898	0.39367
22	0.92840	0.02081	16.82838	0.33663
23	5.61597	0.02869	33.88646	0.30049
24	4.71603	0.00325	23.44861	0.07558
25	7.11337	-0.00143	29.17253	-0.03066
26	7.21593	-0.01344	19.43361	-0.28456

BARTLETTS STATISTIC

30.91

CHI SQUARE STATISTIC AT .05 LEVEL

37.7

ACCEPT THE HYPOTHESIS OF HOMOGENEITY OF VARIANCES

DATA FOR MAGNESIUM

ENGINE NO.	ALFA	BETA	VARIANCE	RHO
1	1.27106	0.00739	0.38782	0.64673
2	0.51338	-0.00203	0.22000	-0.34639
3	4.02446	0.00197	0.28629	0.28962
4	2.59975	-0.00787	0.39171	-0.77457
5	2.27420	0.00840	0.88063	0.66619
6	2.96652	-0.00121	3.11676	-0.06816
7	0.58384	0.00522	1.44880	0.39353
8	4.45159	-0.00715	1.42679	-0.36491
9	1.63911	0.00451	2.23991	0.35906
10	0.58457	-0.00021	0.31716	-0.03084
11	1.27163	0.00512	0.67668	0.40886
12	1.16667	0.00094	0.94859	0.03850
13	3.86771	0.02281	2.31024	0.84098
14	2.89861	0.00057	1.99392	0.05512
15	0.29080	0.02112	0.52083	0.71935
16	1.66836	-0.00554	0.76587	-0.58990
17	2.04964	0.01263	0.24931	0.83256
18	1.32456	0.00409	1.04165	0.43995
19	0.84127	0.00396	0.51687	0.53787
20	0.50504	0.00237	0.93411	0.25669
21	2.41657	0.00093	0.66119	0.09060
22	0.46174	0.01080	0.67101	0.68058
23	0.66353	0.01223	2.05501	0.47893
24	1.39198	-0.00162	0.44549	-0.26511
25	3.70430	-0.00276	2.01043	-0.21981
26	2.22691	-0.00869	0.76340	-0.69563

BARTLETTS STATISTIC

46.66

CHI SQUARE STATISTIC AT .05 LEVEL

37.7

REJECT THE HYPOTHESIS OF HOMOGENEITY OF VARIANCES

DATA FOR NICKEL

ENGINE NO.	ALFA	BETA	VARIANCE	RHO
1	0.28900	0.01329	4.09743	0.42487
2	-0.00302	0.00387	0.55725	0.40512
3	1.27335	-0.00083	1.47447	-0.05634
4	1.53143	-0.00261	1.74781	-0.18892
5	0.68649	0.01023	0.77005	0.75838
6	1.80414	0.00007	1.73732	0.00537
7	3.00174	-0.00079	2.40668	-0.04995
8	2.68564	-0.00787	2.06767	-0.33742
9	0.50566	0.00704	2.74629	0.47708
10	0.04801	0.01028	3.33255	0.41644
11	1.54203	-0.00175	1.23409	-0.11281
12	1.21385	-0.00602	0.94202	-0.24079
13	1.37346	0.00835	1.14372	0.62880
14	2.34792	-0.00399	1.64065	-0.38975
15	-1.48618	0.02354	0.73462	0.69597
16	2.87255	-0.00798	1.28040	-0.63089
17	2.07444	-0.00484	0.56314	-0.35782
18	0.80567	0.00285	0.54576	0.42587
19	0.97008	0.00015	0.88859	0.01831
20	1.22195	0.00373	1.63718	0.30075
21	3.03027	-0.00404	1.51671	-0.28009
22	1.29714	0.00122	0.65921	0.10572
23	1.90540	-0.00371	2.27705	-0.15531
24	0.11514	0.00766	0.89592	0.67501
25	2.32527	-0.00057	7.69561	-0.02387
26	1.13216	-0.00246	0.85947	-0.24979

BARTLETTS STATISTIC

42.43

CHI SQUARE STATISTIC AT .05 LEVEL

37.7

REJECT THE HYPOTHESIS OF HOMOGENEITY OF VARIANCES

DATA FOR SILICON

ENGINE NO.	ALFA	BETA	VARIANCE	RHO
1	0.97502	0.02269	5.61894	0.56473
2	3.56379	0.00047	1.31088	0.03513
3	5.08874	0.00020	1.14555	0.01565
4	3.37554	-0.00402	1.43009	-0.31110
5	9.54492	0.00392	95.18030	0.04004
6	2.75572	0.00857	8.90288	0.27589
7	2.99024	0.00626	2.60247	0.35762
8	3.12190	0.00500	5.37198	0.13986
9	5.23468	-0.00054	7.83646	-0.02473
10	14.02483	-0.05174	54.30548	-0.49574
11	2.45637	0.00851	2.77044	0.34544
12	6.24862	0.00145	18.00914	0.01363
13	3.48370	0.00775	16.04675	0.19656
14	4.74273	-0.00097	1.43503	-0.10929
15	2.12191	0.02054	0.90045	0.60802
16	3.54305	-0.00246	2.20525	-0.18763
17	1.70598	0.01674	0.49007	0.81773
18	3.12749	0.00561	1.89773	0.44513
19	3.28432	0.00311	10.41553	0.11099
20	2.01364	0.00472	3.13833	0.27742
21	4.27623	0.00347	1.21687	0.26992
22	2.61480	0.00685	1.68334	0.34890
23	2.22112	0.01514	3.87579	0.44118
24	2.34076	0.00553	4.85940	0.27276
25	4.16613	0.00015	3.94967	0.00901
26	3.82401	-0.00690	3.46491	-0.33962

BARTLETTS STATISTIC

158.51

CHI SQUARE STATISTIC AT .05 LEVEL

37.7

REJECT THE HYPOTHESIS OF HOMOGENEITY OF VARIANCES

APPENDIX H

GENERAL RESULTS FOR JET ENGINES

Essentially the same analysis was conducted for jet models J52-P-6, J60-P-6, and J33-A-24, as was conducted for model R2000-6. When scatter diagrams for the jet models were plotted they provided no encouragement that a functional relationship might exist between the independent and dependent variables. As part of the analytical analysis a stepwise regression was executed in order to determine which of the two variables, hours since engine oil change or hours since engine overhaul, was the better predictor. For all three models the results were inconclusive. Approximately one-half the engines showed hours since engine oil change to be the better predictor and for the other half hours since engine overhaul was the better. Simple regressions were executed using each of the two independent variables. Regardless of which variable was used in the equation, about half of the regression coefficients were negative. This meant that in about half of the jet engines there was a decrease in the estimate of the density of wear metals as time since oil change increased. Inconsistent behavior such as this could only be interpreted as evidence that neither of the two control variables possessed reliable predictive powers for the dependent variable.

For each model Bartlett's test for homogeneity of variances was conducted. The results were similar to those obtained for the reciprocating engine; i.e., the jet engines of a particular model could not be considered as coming from the same population.

Therefore, as with the reciprocating engines, data manipulation must continue to be carried out separately for each engine.

When the raw data for jet engines was compared against raw data for a reciprocating engine an immediate difference was detected in the densities of all elements. The densities in parts per million for jet engines was, in general, considerably less than that for a reciprocating engine. A typical comparison is shown in Table II. For model J33-A-24 five engines were analyzed, for model J52-P-6 twenty-four engines were analyzed, and for model J60-P-6 twenty-one engines were analyzed. No engine was included in the analysis unless it had eight or more data points. Since the estimated densities for jet models are of the magnitude shown, it is entirely possible that these densities are within the noise level of the spectrometer. This probably explains the fact that neither hours since engine oil change nor hours since engine overhaul appear to be of value in predicting the internal condition of engines chosen from the set of three jet models analyzed in this study.

	Density of Element (PPM) by Model			
Element	R2000-6	J60-P-6	J52-P-6	J33-A-24
Aluminum	7-25	0-2	0-5	0-4
Iron	58-100	1-5	3-6	1-3
Chromium	3-18	1-7	1-9	1-8
Silver	5-11	0-1	0-2	0-3
Copper	9-26	0-1	0-1	0-2
Magnesium	2-4	0-4	0-4	0-1
Nickel	0-6	0-2	0-2	0-7

TABLE II

Comparison of Typical Densities Between Jet
And Reciprocating Engines

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13. ABSTRACT

The spectrometric oil analysis program as applied to naval aviation was developed as a portion of the overall aviation safety program of the U. S. Navy. The equipment and techniques have been refined, and the program has been steadily expanded since its inception in 1955. The value of this system in determining densities of microscopic particles of certain oil-wetted wear metals in samples of oil extracted from aircraft engines has proved to be helpful in predicting incipient engine failure. In this study data relating to both reciprocating and jet engine models was analyzed in an attempt to determine which of the following elements provided significant information regarding the internal condition of the engine: aluminum, iron, chromium, silver, magnesium, nickel, copper, and silicon.

Multiple and simple linear regression analyses and correlation techniques were applied in order to determine the mathematical model which corresponded most closely to the data compiled.

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